ON TESTING A SET OF CORRELATION COEFFICIENTS FOR EQUALITY

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0. Summary. The problem of testing a set of correlation coefficients for equality is discussed. We first generalize a result of Anderson and then provide a criterion of large-sample χ^2 type.

1. The generalization of a result of Anderson. In his preceding paper Anderson [1] has discussed (see Section 5 and Appendix A) the hypothesis that the $p - 1$ smallest latent roots of a correlation matrix of order p are all equal to some unknown value λ, which is equivalent to the hypothesis that the variates are equally correlated with correlation coefficient ρ, where $\lambda = 1 - \rho$. The set of p variates is assumed to follow a multivariate normal distribution.

Let r_{ij} $(i, j = 1, 2, \cdots, p)$ be the sample correlation coefficient between the ith and jth variates found from a random sample of size $n + 1$. Then, adopting Anderson's procedure, we put $y_{ij} = (r_{ij} - \rho)n^{\frac{1}{2}}$ $(i \neq j)$. Taking $i < j$, we have a set of $\frac{1}{2}p(p - 1)$ variates which are asymptotically normally distributed such that

$$\begin{aligned} E(y_{ij}^2) &= \lambda^2(1 + \rho)^2, \\ E(y_{ij}y_{ik}) &= \tfrac{1}{2}\lambda^2\rho(2 + 3\rho) \qquad (j \neq k), \\ E(y_{ij}y_{hk}) &= 2\lambda^2\rho^2 \qquad \text{(no subscripts equal)}. \end{aligned}$$

The results obtained by Anderson show that Bartlett's criterion [2] for testing the hypothesis is asymptotically equal to

$$(1.1) \quad (1/2\lambda^2)\{\sum y_{ij}^2 - (2/p) \sum y_{ij}y_{ik} + [(p - 2)/p^2(p - 1)](\sum y_{ij})^2\},$$

where the summations are over all pairs of unequal suffices.

For the case where $p = 3$ Anderson has proved that the above expression is distributed asymptotically as $(1 - \lambda^2/3)\chi_2^2$, where χ_r^2 denotes a χ^2 variate with r degrees of freedom. A knowledge of this result gave me the idea of generalizing it for any value of p.

In the ensuing algebra it will be convenient to write

$$Y_k = \sum_i y_{ik}, \qquad \bar{y}_k = Y_k/(p - 1),$$

$$Y = \sum_{i<j} y_{ij}, \qquad \bar{y} = 2Y/\{p(p - 1)\},$$

$$y_{(ij)} = y_{ij} - [(p - 1)/(p - 2)](\bar{y}_i + \bar{y}_j) + [p/(p - 2)]\bar{y}.$$

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