ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Eastern Regional Meeting, Cambridge, Massachusetts, May 6–7, 1963. Additional abstracts appeared in the March, 1963 issue.)

6. Asymptotic Joint Distribution of Quantiles From a Bivariate Distribution.

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Let $(X_i, Y_i), i = 1, 2, \cdots, n$ be a random sample of size n from an absolutely continuous d.f. $F(x, y)$, with joint p.d.f. $f(x, y)$. Let $Z_1^{(n)} \leqq Z_2^{(n)} \leqq \cdots \leqq Z_n^{(n)}$ and $W_1^{(n)} \leqq W_2^{(n)} \leqq \cdots \leqq W_n^{(n)}$ be the ordered values of $X_1, \cdots, X_n$ and $Y_1, \cdots, Y_n$ respectively. Further, let ξ_α, η_β be the unique real numbers satisfying $F_1(\xi_\alpha) = \alpha$, $F_2(\eta_\beta) = \beta$, where $F_1(x)$ and $F_2(y)$ are the marginal distribution functions of X and Y (with marginal p.d.f.'s $f_1(x)$ and $f_2(y)$) and $f_1(\xi_\alpha) > 0$, $f_2(\eta_\beta) > 0$. Also let r_n and s_n be two sequences of positive integers such that $\lim_{n\to\infty} r_n/n = \alpha$, $\lim_{n\to\infty} s_n/n = \beta$, $0 < \alpha < 1$, $0 < \beta < 1$, so that $Z_{r_n}^{(n)}$ and $W_{s_n}^{(n)}$ are sample quantiles of order α and β.

If

$$\Sigma = \begin{pmatrix} \alpha(1-\alpha) & \alpha_\beta - q_1 \\ \alpha_\beta - q_1 & \beta(1-\beta) \end{pmatrix},$$

where $q_1 = F(\xi_\alpha, \eta_\beta)$ is non-singular, then the limiting joint distribution of the r.v.'s $n(Z_{r_n}^{(n)} - \xi_\alpha) f_1(\xi_\alpha)$, $n(W_{s_n}^{(n)} - \eta_\beta) f_2(\eta_\beta)$ is a bivariate normal distribution with mean vector 0 and variance covariance matrix Σ. We note that $Z_{r_n}^{(n)}$, $W_{s_n}^{(n)}$ are asymptotically independent if and only if $F(\xi_\alpha, \eta_\beta) = q_1 = \alpha\beta = F_1(\xi_\alpha) F_2(\eta_\beta)$. This result has obvious generalizations.

7. On the Pessimum Interference With Random Signals (Preliminary report).

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For a communication channel accepting as input one real number per unit time of variance $\leqq P$, adding to it independent normal noise of variance N and interference of variance $\leqq J$ which may depend on the signal (sequence of $n \to \infty$ numbers representing one of $M \to \infty$ messages) being sent and the $M - 1$ alternative signals, the channel capacity can be approached by selecting the M signals randomly from among the vectors of length $(nP)^{\frac{1}{2}}$ for sufficiently large N (*Trans. Information Theory* IT-8 (1962), 48–55 and S53–S57). The proof depends on establishing that the worst interference with such random signals is either (1) half the difference between the transmitted signal and a nearest alternative signal or (2) that negative fraction of the transmitted signal plus additional noise which minimizes the effective signal-to-noise ratio. To show no other sort of interference can do worse, we suppose J too small for (1), and (2) unlikely to cause errors. Consequently, the caps which the faces of the Dirichlet region of the transmitted signal cut from the "noise sphere" of radius $(nN)^{\frac{1}{2}}$ surrounding the sum of the transmitted signal plus interference have radii $< \frac{1}{2}\pi - \epsilon$, and no more than a bounded number of them cover any point of the noise sphere (*Ann. Math. Statist.* **32** (1961) 916). Hence (N. M. Blachman and L. Few, "Multiple Packing of Caps on a Sphere", *Mathematika* **10** (1963)), only a vanishing fraction of the surface of the noise sphere lies outside the Dirichlet region, and the error probability $\to 0$.