SAMPLING VARIANCES OF THE ESTIMATES OF VARIANCE COMPONENTS IN THE UNBALANCED 3-WAY NESTED CLASSIFICATION

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1. Introduction. Sampling variances of estimates of components of variance obtained from data, that are unbalanced, are difficult to obtain compared with similar derivation when the data is balanced. Matrix methods of deriving expressions for the sampling variances of the variance component estimates for the unbalanced case are developed in [3] and they are applied to some special cases in [3], [4] and [5]. Here we extend those results to the case of 3-way hierarchial (nested) classification.

2. Model and analysis of variance. In the earlier work [5] the sampling variances of variance component estimates are obtained by Henderson's Method 1 [2] from data having unequal subclass numbers, assuming the completely random model, namely Eisenhart's Model II, [1]. Here we consider the same situation for the 3-way nested classification.

The linear model for an observation x_{ijlm} is

$$x_{ijlm} = \mu + a_i + b_{ij} + c_{ijl} + e_{ijlm}$$

where μ is the general mean, a_i is the effect due to the ith first stage class A_i, b_{ij} is the effect due to the jth second stage class B_{ij} within A_i, c_{ijl} is the effect of lth third stage class C_{ijl} within B_{ij}, and e_{ijlm} is the residual error of the observation x_{ijlm}. We assume the number of first stage classes A_i is α so that $i = 1, \cdots, \alpha$. Within each A-class A_i there are β_i B-classes so that $j = 1, \cdots, \beta_i$. Further within each B_{ij} class there are γ_{ij} C-classes so that $l = 1, \cdots, \gamma_{ij}$. The number of observations in the third stage class C_{ijl} is n_{ijl}. All terms of the model (except μ) are assumed to be independent and normally distributed random variables with zero means and variances σ_α^2, σ_β^2, σ_γ^2 and σ_e^2 respectively. These are the variance components which are to be estimated. The sampling variances of these estimates are to be found.

The usual analysis of variance is given in Table I where $\beta = \sum_i \beta_i$, $\gamma = \sum_i \sum_j \gamma_{ij}$, $N = \sum_i \sum_j \sum_l n_{ijl}$ and with usual notation for totals and means.

The components of variance are estimated by equating each sum of squares of the ANOVA (except for "total") to its expected value. Denoting the resulting

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