ON A MODIFICATION OF CERTAIN RANK TESTS

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1. Introduction. This note is concerned with a modification of certain well-known rank tests based on the theory of Chernoff-Savage [2]. Let $X_1, X_2, \cdots, X_m$ and $Y_1, Y_2, \cdots, Y_n$ be two independent samples from populations with continuous distribution functions F and G. Our interest is to test the hypothesis $H_0:F \equiv G$. For the problem for location, we assume $G(x) = F(x + \theta)$ and want to test $H_0:\theta = 0$ against the alternative $H_1:\theta > 0$. On the other hand, the problem for scale is to test $H_0:\theta = 1$ against $H_2:\theta > 1$ under the restrictions $G(x) = F(x/\theta)$ and $F(0) = \frac{1}{2}$. There are well-known test statistics, Wilcoxon's U [4] for the location problem and Mood's M [3] and Ansari-Bradley's S [1] for the scale problem. We shall modify these statistics to raise the asymptotic efficiency of the corresponding tests.

2. Test statistics. We adopt the following statistics for $k > 0$,

$$mU_k = \sum_{i=1}^{N} (i/N)^k Z_i, \qquad N = m + n, \tag{1}$$

$$mS_k = \sum_{i=1}^{p} (i/N)^k Z_i + \sum_{i=p+1}^{N} \{(N + 1 - i)/N\}^k Z_i, \quad p = [\tfrac{1}{2}(N + 1)], \tag{2}$$

$$mM_k = \sum_{i=1}^{N} |i/N - (N + 1)/2N|^k Z_i, \tag{3}$$

where Z_i is 1(0) if the ith smallest among the combined sample is $X(Y)$. We may directly apply the theorem of Chernoff-Savage for U_k and S_k and show their asymptotic normality with the mean $\mu_k(\theta)$ and the variance $\sigma_k^2(\theta)$,

$$\begin{aligned} \mu_k(\theta) &= \int_{-\infty}^{\infty} H(x)^k \, dF(x), && \text{for } U_k \\ &= \int_{-\infty}^{0} H(x)^k \, dF(x) + \int_0^{\infty} \{1 - H(x)\}^k \, dF(x), && \text{for } S_k \end{aligned} \tag{4}$$

and under the hypothesis H_0

$$\begin{aligned} N\sigma_k^2 &= [(1 - \lambda_N)/\lambda_N]k^2/(2k + 1)(k + 1)^2, && \text{for } U_k, \\ &= [(1 - \lambda_N)/\lambda_N]k^2/4^k(2k + 1)(k + 1)^2, && \text{for } S_k, \end{aligned} \tag{5}$$

where $H(x)$ is the combined population distribution function and $\lambda_N = m/N$. Though the assumption (4) of their theorem (see [2]) is not satisfied for M_k, we may similarly prove its asymptotic normality from the fact that the following

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