ON THE LIMIT BEHAVIOUR OF EXTREME ORDER STATISTICS[1]

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0. Introduction. This paper is concerned with some recent developments in the theory of limit behaviour of extreme order statistics. Most of the paper is devoted to the discussion of limit distributions and of stability properties for order statistics of independent random variables. While the situation here is already well explored, very little is known in the case of dependent random variables. Those results that are known for the dependent case have been obtained in the last few years.

In Section 1 we introduce notations and definitions and state a few elementary facts concerning the distributions of the set of order statistics corresponding to a set of independent, identically distributed random variables. Throughout Sections 2, 3 and 4 we assume independence of the basic variables. In Section 2 we deal with limit distributions, while in Sections 3 and 4 we deal with stability in probability and stability almost surely. Finally, in Section 5 we turn to the dependence case, summarizing some recent results due mainly to Berman [3], [4] and [5].

Our principal result is contained in Section 4; it is a proof of the sufficiency of a simple condition for stability almost surely of the maximal order statistic. That condition was introduced and studied by Geffroy [7].

The aim in writing this paper has been twofold: to give a brief summary of the current state of research for the topic in question and to indicate certain recent contributions to this topic due to the author. A number of contributions are not mentioned; these are noted in the references.

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1. Preliminaries. Let $X_1, X_2, \cdots, X_n, \cdots$, be a sequence of random variables defined on a probability field $(\Omega, \mathcal{A}, P)$. To any set $X_1, X_2, \cdots, X_n$ $(n = 1, 2, \cdots)$ let $X_{n1}, X_{n2}, \cdots, X_{nn}$ denote the corresponding set of order statistics, where for all points $\omega \in \Omega$, $X_{nk}(\omega)$ is equal to the kth largest of the values $X_1(\omega), X_2(\omega), \cdots, X_n(\omega)$. Thus $X_{n1} \geqq X_{n2} \geqq \cdots \geqq X_{nn}$. Throughout the paper k will denote a fixed positive integer. We will study the limit behaviour of sequences $\{X_{nk}\}$ of extreme upper order statistics. Obviously the results we mention hold with trivial modifications for sequences $\{X_{n,n-k}\}$ of extreme lower order statistics.

In the sequel, unless otherwise explicitly stated, we assume the variables

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