EXISTENCE OF BOUNDED LENGTH CONFIDENCE INTERVALS[1]

By Rajinder Singh

University of Illinois and Punjab University, Chandigarh

0. Introduction. Let X be a random variable whose distribution P belongs to a family of distributions $\mathcal{P}$. Let h be a real valued function defined on $\mathcal{P}$. This paper is concerned with the existence of estimates of prescribed accuracy for such functions h based on observations on X under different types of sampling plans. By an estimate of prescribed accuracy we mean a confidence interval of prescribed length and confidence coefficient, or a point estimate with prescribed expected loss. The sampling plans considered here are the m-stage, $m \geqq 1$, and sequential sampling plans. It may be pointed out that estimates with prescribed accuracy have been defined in the literature in various ways, [2], [3], [4], [6], [7], [9].

In most problems of estimation, estimates based on samples of fixed sizes have precisions which depend on unknown parameters. Consequently estimates with prescribed accuracy are not available without resort to multistage and sequential sampling plans. In fact, [1] in many non-parametric problems, even sequential sampling plans fail to give estimates with prescribed accuracy. It therefore becomes desirable to know whether, in a given problem of estimation, estimates of prescribed accuracy for the functions h exist under a given type of sampling plan.

It is shown that if h has a bounded length confidence interval based on one-stage sampling plans then h is uniformly continuous on $(\mathcal{P}, d^1)$, and if h has a bounded length confidence interval based on m-stage or sequential sampling plans, then h is continuous on $(\mathcal{P}, d^1)$, where d^1 is the familiar absolute variational distance on $\mathcal{P}$.

Further, if g is a uniformly continuous function of a real variable and h has a bounded length confidence interval based on m-stage sampling plans, then the composite function $g(h)$ has also a bounded length confidence interval based on m-stage sampling plans. If g is simply continuous (but not uniformly so), $g(h)$ has a bounded length confidence interval based on $2m$-stage sampling plans.

1. Definition, notation and statement of the problem. Fixed throughout are Ω, $\mathcal{A}$, $\mathcal{P}$ where $\mathcal{P}$ is a family of probability measures on $\mathcal{A}$, a σ-field of subsets of the set Ω; $X_1, X_2, \cdots$, are random variables such that, for each P in $\mathcal{P}$, $X_1, X_2, \cdots$ are independent and identically distributed. Point sets $\{\omega: Y(\omega) \varepsilon B\}$, where Y is a random variable and B is a Borel set, will be denoted by $(Y \varepsilon B)$

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