THE RELATION BETWEEN PITMAN'S ASYMPTOTIC RELATIVE EFFICIENCY OF TWO TESTS AND THE CORRELATION COEFFICIENT BETWEEN THEIR TEST STATISTICS[1]

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1. Introduction. It is well known (cf. e.g., Cramér [1], pp. 477–483) that, under certain regularity conditions, the efficiency of an unbiased estimate of a parameter relative to an efficient unbiased estimate is equal to the square of the correlation coefficient between the two estimates.

In this paper the relation between Pitman's asymptotic relative efficiency $e(T', T)$, of a test T' with respect to a test T, and the correlation coefficient between their test statistics, t' and t, will be considered.

If the test statistic of T is an efficient estimate of the underlying parameter and $g(t')$ is a consistent estimate of this parameter, then Noether [10] proved (cf. also Kendall and Stuart [6], Section 25.13 and Stuart [13]) that, under certain regularity conditions, the limiting correlation coefficient between t and $g(t')$ equals $[e(T', T]^{\frac{1}{2}}$. This result was used by Stuart ([14] and [15]) to find the asymptotic relative efficiency of several nonparametric tests for normal alternatives.

In this paper it will be shown that, under certain regularity conditions, the statistic t' itself has limiting correlation coefficient $[e(T', T)]^{\frac{1}{2}}$ with the test statistic t of a (in a later to be defined sense) best test T.

For the case of the two sample location problem Hájek [4] proved this relation between $[e(T', T)]^{\frac{1}{2}}$ and $\rho(t', t)$ for rank-orders tests. In his case T' (respectively T) is the locally most powerful rank-order test for testing $\theta = 0$ if the two samples are from distributions with distribution functions $F(x)$ and $F(x + \theta)$ (respectively $G(x)$ and $G(x + \theta)$).

The theorem will be stated and proved in Section 2; Section 3 contains some examples.

2. The theorem. Let T and T' be two tests for the hypothesis H_0: $\theta = \theta_0$ against the alternative $\theta > \theta_0$. Then the relative efficiency of T' with respect to T is the ratio n/n', where n and n' are the number of observations necessary to give T and T' the same power β for a given level of significance α. The concept of asymptotic relative efficiency is due to Pitman [11]. He considers the limit of n/n' for a sequence of alternatives depending on the sample size and converging to H_0 in such a way that the power of both tests converges to a limit <1. Pitman proved the following theorem (cf. e.g., Noether [9] and [10]).

Received March 19, 1963.

[1] This research was supported in part by the National Science Foundation under Grant Number G-19126 and by the National Institutes of Health under Grant Number 2G-43 (7 + 8).