# THE LIMITING POWER OF CATEGORICAL DATA CHI-SQUARE TESTS ANALOGOUS TO NORMAL ANALYSIS OF VARIANCE[1]

By Earl L. Diamond

*The Johns Hopkins University*

**1. Introduction.** Mitra [5] has derived the Pitman limiting power [10] of the frequency chi-square test. He considers the case in which it is hypothesized that the cell probabilities are specified functions of unknown parameters which are to be estimated from the sample. For completeness and because it is needed for subsequent proofs, his theorem is presented without proof. In the usual situation of making tests for categorical data tables of two or more dimensions, where the hypotheses to be tested are of the forms presented by Roy and Mitra [12] and Diamond, Mitra and Roy [3], the hypothesis is that the cell probabilities are specified functions of unknown parameters which are to be estimated from the sample and which are subject to specified functional conditions. In Section 3 of this paper a theorem covering this latter case is presented without proof. The proof which is exactly analogous to that for Mitra's theorem covering the simpler case is presented elsewhere by Mitra [6], Ogawa [7] and Diamond [2].

Another type of test, analogous to those of normal analysis of variance, might be considered. One assumes that the cell probabilities are specified functions of unknown parameters. This assumption, together with the initial sampling distribution, form the "model". The hypothesis to be tested is that the parameters satisfy specified functional relationships. In Section 4 two theorems are proved. The first, a necessary preliminary, covers the situation in which the hypothesis is that some of the unknown parameters have specified values. The second theorem in Section 4 covers the situation in which the hypothesis is that the parameters satisfy specified functional relationships. In both theorems, the limiting distribution is shown to be a non-central chi-square with certain degrees of freedom and a specific non-centrality parameter in the non-null case.

**2. Mitra's theorem in frequency chi-square** [5]. Suppose we have $R = \sum_{i=1}^{q} r_i$ functions $p_{ij}(\alpha_1, \alpha_2, \cdots \alpha_s)$ $(i = 1, 2, \cdots, q; j = 1, 2, \cdots, r_i)$ of $s < R - q$ parameters $\alpha_1, \alpha_2, \cdots, \alpha_s$ such that for all points of a nondegenerate interval $A$ in the $s$-dimensional space of the $\alpha_k$'s the $p_{ij}$ satisfy the following conditions:

(a) $\sum_{j=1}^{r_i} p_{ij}(\alpha_1, \alpha_2, \cdots, \alpha_s) = 1$ for $i = 1, 2, \cdots, q$,

(b) $p_{ij}(\alpha_1, \alpha_2, \cdots, \alpha_s) > c^2 > 0$ for all $ij$,

(c) Every $p_{ij}$ has continuous derivatives $\partial p_{ij}/\partial \alpha_k$ and $\partial^2 p_{ij}/\partial \alpha_k \partial \alpha_l$,

(d) The $R \times s$ matrix $\{\partial p_{ij}/\partial \alpha_k\}$ is of rank $s$.

(It is assumed that the index pairs $(i, j)$ indicating the rows of the above matrix

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