A PROPERTY OF THE METHOD OF STEEPEST ASCENT

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1. Introduction. Box and Wilson (1951) proposed the method of steepest ascent as part of a general technique for attaining optimal operating conditions. The experimental points are coded and the path of steepest ascent is calculated in the space of the coded points. This path is decoded and further experiments are performed along it until it is felt that no further appreciable gain can be realized. It is well known that the path of steepest ascent is not scale invariant. In general, the decoded path and the path of steepest ascent calculated from the uncoded points, are distinct.

This procedure of moving along the gradient of a fitted plane has appeal since we can expect a plane to provide a reasonable fit of the surface locally. It would seem that we might gain efficiency by revising the path after each observation. Therefore, suppose that as each additional experiment is performed, it is combined with all previous experiments to calculate a revised path by fitting a plane. The results of Plackett (1950) for inclusion of additional observations, have been put in a form useful for this purpose by Box and Wilson (1951). For certain designs, if an additional point is taken along the path of steepest ascent in the space of the coded variables, the revised path is the same as the initial path. In the space of the uncoded variables, if an additional point is taken along the decoded path, the revised path of steepest ascent is the same as the initial path. Indeed, these statements remain true however many additional experiments are performed.

2. Notation. Let X_1 be the $N_1 \times (p + 1)$ initial design matrix; X_2 be the $N_2 \times (p + 1)$ design matrix of the N_2 additional points, $B' = [b_0, b_1, \cdots b_p]$ be the vector of coefficients of the plane fitted to the initial N_1 responses; $A' = [a_0, a_1, \cdots, a_p]$ be the vector of coefficients of the plane fitted to the $N_1 + N_2$ responses, and Y_2 be the $N_2 \times 1$ vector of responses at the N_2 additional points. With this notation the expression for A, (see Box and Wilson) is written

$$A = B + J'G\Delta \tag{1}$$

where

$$J = X_2(X_1'X_1)^{-1}, \quad G = (I_{N_2} + JX_2'), \quad \Delta = Y_2 - X_2B.$$

3. Results. Consider the three conditions:

$$X_2 = (X_{20}, X_{21}) = (X_{20}, HB_1'T^2) \tag{a}$$

Where $X_{20} = \text{col}\ (1, \cdots, 1)$, $H = \text{col}\ (h_1, \cdots, h_{N_2})$, $B_1 = \text{col}\ (b_1, b_2, \cdots, b_p)$, and $T = \text{diag}\ (t_1, \cdots, t_p)$.

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