A REVIEW OF THE LITERATURE ON A CLASS OF COVERAGE PROBLEMS

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0. Introduction. In recent years a large number of publications have appeared on probability problems arising from ballistic applications. Many of these papers and reports are concerned with topics which are often referred to as coverage problems. Some of the results are found only in obscure sources which are not readily available. Consequently it is a difficult and time consuming task to locate the numerous publications on this subject, a fact that has led to considerable duplication of effort and waste of time. It is hoped that this review will improve the situation and be of some use to those who have an interest in problems of this type.

The discussion of a simple example will serve the purpose of introducing some of the ideas and language needed for the definition of coverage problems which is presented later in the introduction. Suppose that a point target is located at the origin of a two dimensional coordinate system. A weapon with killing radius R is aimed at the origin with the intention of destroying the point target. When the weapon arrives at the target, the latter is located at (x_1', x_2'), a randomly selected position within or on a circle of radius D. That is, the probability density function of (x_1', x_2') is $g(x_1', x_2') = (\pi D^2)^{-1}$, $0 \leqq x_1'^2 + x_2'^2 \leqq D^2$. Assume that aiming errors are circularly normally distributed with unit variance so that the center of the lethal circle, (x_1, x_2), has p.d.f.,

$$f(x_1, x_2) = (2\pi)^{-1} \exp[-\tfrac{1}{2}(x_1^2 + x_2^2)].$$

Now a given point (x_1', x_2') will be destroyed if the impact point of the weapon is within R units of (x_1', x_2'). The probability that this happens is

$$h(x_1', x_2') = \iint\limits_{(x_1 - x_1')^2 + (x_2 - x_2')^2 \leqq R^2} f(x_1, x_2)\, dx_1\, dx_2 .$$

The probability of destroying the target (that is, the probability that the impact point is within R units of the target given that the target is as likely to be at one point as any other in a circle of radius D) is

$$P(R, D) = \iint\limits_{x_1'^2 + x_2'^2 \leqq D^2} h(x_1', x_2') g(x_1', x_2')\, dx_1'\, dx_2' .$$

The evaluation of this integral will be discussed in Paragraph 2.2.

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