

- (1) M. O. LOCKS, M. J. ALEXANDER and B. J. BYARS, *New Tables of the Non-central  $t$  Distribution*. ARL 63-19, Aeronautical Research Laboratories, Wright-Patterson Air Force Base, January, 1963. For sale by OTS at \$6.00 v + 463 pp. of which 11 pp. are nontabular.
- (2) D. B. OWEN, *Factors for One-Sided Tolerance Limits and for Variables Sampling Plans*. SCR-607, Sandia Corporation, March, 1963. For sale by OTS at \$5.00 412 pp. of which 64 pp. are nontabular.

Review by B. L. WELCH

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The above two volumes provide new tabulations for the non-central  $t$ -distribution. The non-central  $t$ -statistic is defined by the ratio of  $(z + \delta)$  to  $\sqrt{w}$  where  $z$  is a unit normal deviate,  $w$  is distributed as  $\chi^2/f$  and  $\delta$  is the non-centrality parameter. The distribution of  $t$  is required in the solution of a wide variety of problems which arise in sampling from a normal distribution. Early tables of the cumulative distribution of  $t$  were calculated for one restricted purpose by J. Neyman (1935) and J. Neyman and B. Tokarska (1936), viz. to facilitate the calculation of the power function of the standard "Student"  $t$ -test. For other purposes a wider coverage of the parameters involved is necessary and an approximation useful for all values, excluding only small  $f$ , was given by W. J. Jennett and B. L. Welch (1939). This approximation stems from the fact that  $\sqrt{w}$  is nearly normally distributed and that the mean and standard deviation of  $\sqrt{w}$  are  $a$  and  $b(2f)^{-\frac{1}{2}}$ , respectively, where  $a$  and  $b$  differ from unity by quantities of order  $f^{-1}$ . Taking  $\sqrt{w}$  to be *exactly* normally distributed with mean unity and standard deviation  $(2f)^{-\frac{1}{2}}$  we can indeed deduce the following approximate formulae connecting  $\delta$  with the percentage points  $t_\epsilon$  of the non-central  $t$ -distribution:

$$\delta = t_\epsilon - K_\epsilon(1 + t_\epsilon^2/2f)^{\frac{1}{2}},$$

$$t_\epsilon = [\delta + K_\epsilon(1 + \delta^2/2f - K_\epsilon^2/2f)^{\frac{1}{2}}]/(1 - K_\epsilon^2/2f).$$

(In these formulae  $K_\epsilon$  is the normal deviate exceeded with probability  $\epsilon$ , and  $t_\epsilon$  is defined by the relation  $\Pr(t > t_\epsilon | \delta) = \epsilon$ ).

Using this approximation as a starting point N. L. Johnson and B. L. Welch (1940) developed series expansions permitting a more accurate evaluation of the relation between  $\delta$  and  $t_\epsilon$ , and a tabulation of the relationship for certain values of  $f$  and  $\epsilon$  was carried out by N. L. Johnson. These tables, however, ran only to about 10 *Biometrika* pages and their use involved a fair amount of subsidiary calculation.

The advent of electronic computing has brought about the production of much more extensive tables, the first of these, due to G. J. Resnikoff and G. J. Lieberman (1957), occupying a volume of about 400 pages and being constructed by methods differing completely from those employed in earlier tabulations.