GENERALIZED LEAST SQUARES ESTIMATORS FOR RANDOMIZED FRACTIONAL REPLICATION DESIGNS[1]

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1. Introduction. The problem of choosing a $1/M = 1/2^{m-s}(m > s)$ fractional replication from a factorial system of size $N = 2^m$ and deciding upon appropriate estimator of the parameters characterizing the factorial model (main effects and interactions) has been studied by Dempster [2], [3], Takeuchi [11], [12], Ehrenfeld and Zacks [4], [5], Zacks [13], Shah and Kempthorne [9], [10]. In all these studies the type of estimators considered is that which yields, under a randomized procedure with equal probabilities of choice, unbiased estimates of a specified subvector of parameters, which lies in the range of the design matrix. A *complete* class of linear unbiased estimators for estimating an M-dimensional subvector of pre-assigned parameters, under a randomized fractional replication procedure, was studied in [13]. Optimal procedures of choosing a fractional replication and an estimator were studied in [5].

In the present paper we consider the problem of estimating the *entire* vector of N parameters, on the basis of a fractional replication of size $S = 2^s$. This problem arises when we wish to explore a given factorial system, and find which are the important parameters. Such a problem calls for a sequence of fractional replication designs, where each design is modified by the information attained in previous experiments. At least in the first stages of such a sequence one would like to estimate the entire vector of N parameters. Presently we consider this estimation problem for a one stage fractional replication design. We adopt the least-squares principle, and consider the class of all *generalized least-squares* estimators (g.l.s.e.) which corresponds to a given block of S treatment combinations. The term generalized least-squares estimators is used since the corresponding matrices of the normal equations are singular. The linear spaces of all g.l.s.e. associated with the various fractional replication designs, of the type considered here (see Ehrenfeld and Zacks [4] and Zacks [13]), are characterized in terms of the linear coefficients of the factorial model. As proven, every g.l.s.e. is represented by an M-dimensional vector $(\lambda_0, \cdots, \lambda_{M-1})$ where $\sum_{u=0}^{M-1} \lambda_u = 1$. Some statistical properties of the g.l.s.e., under randomized fractional replication, are studied. First we prove that there is no randomized fractional replication procedure for which an unbiased g.l.s.e. of the entire vector of parameters exists. The problem of which fractional replication to use and which g.l.s.e. to apply is studied in a general decision framework, with the trace of the mean-square-error

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