COMPARISON OF REPLACEMENT POLICIES, AND RENEWAL THEORY IMPLICATIONS

By Richard E. Barlow[1] and Frank Proschan

Boeing Scientific Research Laboratories

1. Introduction and preliminaries. Among the most useful replacement policies currently in popular use are the age replacement policy and the block replacement policy. Under an *age replacement* policy a unit is replaced upon failure or at age T, a specified positive constant, whichever comes first. Under a *block replacement* policy a unit is replaced upon failure and at times T, $2T$, $3T$, $\cdots$. We shall assume for both policies that units fail permanently, independently, and that the time required to perform replacement is negligibly small. Block replacement is easier to administer since the planned replacements occur at regular intervals and so are readily scheduled. This type of policy is commonly used with digital computers and other complex electronic systems. On the other hand, age replacement seems more flexible since under this policy planned replacement takes into account the age of the unit. It is therefore of some interest to compare these two policies with respect to the number of failures, number of planned replacements, and number of removals. ("Removal" refers to both failure replacement and planned replacement.)

Block replacement policies have been investigated by E. L. Welker, 1959, R. F. Drenick, 1960, and B. J. Flehinger, 1962. Age replacement policies have been studied by G. Weiss, 1956, and Barlow and Proschan, 1962, among others.

The results of this paper depend heavily on the properties of distributions with monotone failure rate (Barlow, Marshall, and Proschan, 1963). If a unit failure distribution F has a density f, it can be verified by differentiating $\log \bar{F}(x)$ that the failure rate $r(x) = f(x)/\bar{F}(x)$ is increasing (decreasing) if $\log \bar{F}(x)$ is concave when finite (is convex on $[0, \infty)$). We consistently use $\bar{F}$ for $1 - F$. For mathematical convenience and added generality, we use this concavity (convexity) property as the definition of increasing (decreasing) failure rate whether a density exists or not. We shall refer to increasing failure rate by IFR and decreasing failure rate by DFR. It is also easy to show that F is IFR(DFR) if and only if

$$F_x(\Delta) = [F(x + \Delta) - F(x)]/\bar{F}(x)$$

is increasing (decreasing) for all x such that $\Delta > 0$ and $\bar{F}(x) > 0$. This implies F is IFR(DFR) if and only if

$$\bar{F}(x - \Delta)/\bar{F}(x) \tag{1.1}$$

Received 22 July 1963; revised 18 November 1963.

[1] Now at the University of California, Berkeley.

Institute of Mathematical Statistics is collaborating with JSTOR to digitize, preserve, and extend access to *The Annals of Mathematical Statistics.* www.jstor.org