QUEUES WITH BATCH DEPARTURES II

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1. Introduction; queue size after a departure. In this paper we continue the study of queues with batch departures undertaken by Foster and Nyunt in [4]. For a detailed description of the queueing model we refer to their paper. In brief, we assume units to arrive at the instants of a Poisson process with parameter λ and to be served in batches of fixed size k, the service time distribution $H(x)$ being arbitrary. It will be convenient for later use to take the mean of $H(x)$ as r/μ and to define $\rho = r\lambda/\mu$, the ratio of the mean service time to the mean inter-arrival time. (In [4] the mean of $H(x)$ was taken as $1/\mu$ and ρ was defined as λ/μ, so that basically the definition of ρ is unchanged in the present paper.) The traffic intensity is thus $\tau = \rho/k$ and we assume $\tau < 1$. We also need the Laplace Transform $\psi(s) = \int_0^\infty e^{-sx}\, dH(x)$. Let $\xi(t)$ denote the number of units in the system including the batch undergoing service (if any) at the instant t. We say that the system is in state E_j at the instant t if $\xi(t) = j$. Let

$$p_j^+(n) = P[\xi(\sigma_n + 0) = j]$$

where σ_n denotes the instant of departure of the nth batch. It has been shown in [4] that when $\rho < k$, the limiting probabilities

$$(1) \qquad p_j^+ = \lim_{n\to\infty} p_j^+(n)$$

exist and their generating function is given by

$$(2) \quad P^+(z) = \sum_{j=0}^{\infty} p_j^+ z^j = \frac{\sum_{j=0}^{k-1} p_j^+(z^k - z^j)}{z^k/K(z) - 1} = \frac{(k-\rho)(z-1)\prod_{j=1}^{k-1}\left(\frac{z-\delta_j}{1-\delta_j}\right)}{z^k/K(z) - 1}$$

where $K(z) = \psi\{\lambda(1 - z)\}$ and $1, \delta_1, \delta_2, \cdots \delta_{k-1}$ are the roots of the equation $z^k = K(z)$ on or within the unit circle.

From (2) it follows that

$$\sum_{j=0}^{k-1} p_j^+(z^k - z^j) = (k-\rho)(z-1)\prod_{j=0}^{k-1}(z-\delta_j)/(1-\delta_j)$$

from which we obtain by differentiation,

$$(3) \qquad \sum_{j=0}^{k-1} p_j^+(k-j) = k - \rho.$$

We shall use this result (3) in the sequel.

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