COMPARISON OF THE POWER FUNCTIONS FOR THE TEST OF INDEPENDENCE IN 2 × 2 CONTINGENCY TABLES

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1. Introduction. One of the classical problems in statistical theory is that of testing for independence in 2 × 2 contingency tables. Barnard [1] delineated three distinct experimental situations, [termed the double dichotomy (DD), the 2 × 2 comparative trial (CT), and the 2 × 2 independence trial (IT)] which lead to the presentation of data in the form of 2 × 2 contingency tables. Approximate power functions for tests of independence were considered by Patnaik [10] and Sillitto [12] for the 2 × 2 CT, and Bennett and Hsu [2] have calculated exact values of power for the 2 × 2 IT and 2 × 2 CT, in each case the test used being the conditional test devised concurrently by Yates [14] and Fisher [5]. However, power calculations for the DD have not been made, and hence comparisons of the power functions in the three situations are lacking. This paper represents a contribution in this last direction.

2. Distribution theory. Abstractly, the three experimental situations as outlined by Barnard are describable in the following manner:

I. Double Dichotomy (DD). A total of n similar balls is randomly selected from an urn containing a large number of balls, each ball labeled A_1 or A_2 and also labeled B_1 or B_2. An observed result of the experiment is represented in the form of Table I, where none of the marginal totals are fixed and n_{11} is the observed number of balls labeled A_1 and B_1. It is assumed that the probabilities of occurrence of the various markings of the balls is given by Table II, together with the marginal sums.

II. 2 × 2 *Comparative Trial* (CT). Samples of sizes $n_{1\cdot}$ and $n_{2\cdot}$ are drawn from urns A_1 and A_2 respectively. The numbers of balls labeled B_1 in the two samples (i.e., n_{11} and n_{21}) constitute independent variables with binomial distributions, where the proportion of balls marked B_1 in urn A_i is p_i, $i = 1, 2$. With this type of experiment, one set of marginal totals is fixed in advance as in Table I, namely, $n_{1\cdot}$ and $n_{2\cdot}$.

III. 2 × 2 *Independence Trial* (IT). A total of n similar balls, $n_{1\cdot}$ marked A_1 and $n_{2\cdot}$ marked A_2, are placed in an urn, then withdrawn randomly in order. They are then placed in a row of n cells, of which $n_{\cdot 1}$ have been labeled B_1 and $n_{\cdot 2}$ labeled B_2. The result of the experiment is presented in Table I, where n_{11} is the observed number of balls marked A_1 in receptacles labeled B_1. Both sets of marginal totals are fixed in advanced by the conditions of the experiment.

The probability of observing the sample point $(n_{11}, n_{12}, n_{21}, n_{22})$ in the DD

Received 12 November 1963; revised 12 March 1964.

[1] Research sponsored by the Office of Naval Research.

[2] Research sponsored by the Office of Naval Research and by NSF Grant G 18976.