ESTIMATING THE CURRENT MEAN OF A NORMAL DISTRIBUTION WHICH IS SUBJECTED TO CHANGES IN TIME[1]

By H. Chernoff and S. Zacks

Stanford University and Technion, Israel Institute of Technology

1. Introduction. The present study was motivated by consideration of a "tracking" problem. Observations are taken on the successive positions of an object traveling on a path, and it is desired to estimate its current position. If the path is smooth, regression estimates seem appropriate. If, however, the path is subject to occasional changes in direction, regression will give misleading results long after a naive observer would have made corrections. Our objective is to arrive at a simple formula which implicitly accounts for possible changes in direction and discounts observations taken before the latest change.

To develop insight into the nature of a reasonable procedure, we study a simpler problem. In this problem successive observations are taken on n independently and normally distributed random variables $X_1, X_2, \cdots, X_n$ with means $\mu_1, \mu_2, \cdots, \mu_n$ and variance 1. Each mean μ_i is equal to the preceding mean μ_{i-1} except when an occasional change takes place. The object is to estimate the current mean μ_n.

We shall study this problem from a Bayesian point of view. First, we assume that the time points of change obey an arbitrary specified *a priori* probability distribution appropriate to the special case being studied. Second, we assume that the amounts of change in the means, when changes take place, are independently and normally distributed random variables, with mean 0 and variance σ^2. Third, we assume that the current mean μ_n is a normally distributed random variable with mean 0 and variance τ^2. Letting τ^2 approach infinity, we derive, according to these assumptions, a Bayes estimator of μ_n for an *a priori* uniform distribution on the whole real line and a quadratic loss function. This estimator is invariant under translations of X_i. The minimum variance linear unbiased estimator (MVLU) of μ_n is also derived. The MVLU estimator is considerably simpler than the Bayes estimator. However, when the expected number of changes in the means is neither zero nor $n - 1$ the Bayes estimator is more efficient than the MVLU one. Generally, the Bayes estimator is very difficult for applications, since it requires many involved computations. A considerable simplification is attainable in the formula for the general Bayes estimator by letting the a-priori variance of the changes, σ^2, approach zero. This simplified estimator might not be an efficient one in cases of large changes. As an alternative, we consider the problem where the *a priori* distribution of time points of change

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