# ABSTRACTS OF PAPERS

*(Abstracts of papers presented at the Annual Meeting, Amherst, Massachusetts, August 26–29, 1964. Additional abstracts appeared in earlier issues.)*

**40. Some Properties of the Dependence Capacity of a Stochastic Process.** J. G. Baldwin, Research Triangle Institute.

Let $T$ be an index set and $\hat{\xi} = \{\xi_t \mid t \varepsilon T\}$ be a stochastic process. For every finite set of random variables $\xi_{t_1}, \cdots, \xi_{t_n}$ let $p_{\xi_{t_1} v \cdots v \xi_{t_n}}$ be their joint distribution and $p_{\xi_{t_1}} x \cdots x p_{\xi_{t_n}}$ the correspondingly induced product distribution. We set $U_{\hat{\xi}}(t_1, \cdots, t_n) = \int \beta \, dp_{\xi_{t_1} v \cdots v \xi_{t_n}}$ if $p_{\xi_{t_1} v \cdots v \xi_{t_n}} \ll p_{\xi_{t_1}} x \cdots x p_{\xi_{t_n}}$ ($\beta$ being the Random-Nikodym density) and $U_{\hat{\xi}}(t_1, \cdots, t_n) = \infty$ otherwise. $U_{\hat{\xi}}(t_1, \cdots, t_n)$ vanishes if and only if $\xi_{t_1}, \cdots, \xi_{t_n}$ are independent. $U_{\hat{\xi}}$, considered as a function on the ring $R(T)$ of finite subsets of $T$ (setting $U_{\hat{\xi}}(N) = 0$ if $N$ is empty or contains one element), is a non-negative monotonic increasing function enjoying the property, $U_{\hat{\xi}}(E \cup F) + U_{\hat{\xi}}(E \cap F) \geqq U_{\hat{\xi}}(E) + U_{\hat{\xi}}(F)$, of being a monotone capacity of at least the second order and hence is called the dependence capacity of the process. Among other properties of $U_{\hat{\xi}}$ the following are of particular interest: (1) A necessary and sufficient condition that $\hat{\xi}$ be Markovian ($T$ being a subset of the real line) is that for each set $\{t_1, \cdots, t_n\}$

$$U_{\hat{\xi}}(t_1, \cdots, t_n) = \sum_{k=1}^{(n-1)} U_{\hat{\xi}}(t_k, t_{k+1}).$$

(2) If $\hat{\xi}$ is Markovian, then for every process $\hat{\eta}$ with $U_{\hat{\xi}}(s, t) = U_{\hat{\eta}}(s, t)$ for all $s, t \varepsilon T$, $U_{\hat{\xi}} \leqq U_{\hat{\eta}}$. Hence the quantity $\mu(t_1, \cdots, t_n) = U_{\hat{\eta}}(t_1, \cdots, t_n) - \sum_{k=1}^{n-1} U_{\hat{\eta}}(t_k, t_{k+1})$ is a measure of the "Markovity" of the process $\hat{\eta}$ on the set $\{t_1, \cdots, t_n\}$ and for a sequence of variables $\eta_1, \eta_2, \cdots$ the quantity $\bar{\mu}(\hat{\eta}) = \lim_{t\to\infty} t^{-1}\mu(\{1, \cdots, t\})$ is meaningful as the mean Markovity of the process. (3) If $U_{\hat{\xi}}$ is bounded on $R(T)$ then any two variables $\xi_s, \xi_t$ become asymptotically independent as $|s - t| \to \infty$.

**41. Best Sequential Tests of an Identification Hypothesis in a Degenerate Case** (Preliminary Report). S. Blumenthal, T. Christie and M. Sobel, Rutgers University and University of Minnesota.

Given $k$ populations and two distribution functions having densities $f(x)$ and $g(x)$ respectively, we assume that $k - 1$ populations have density $f(x)$ and one has density $g(x)$. The problem is to identify which population has density $g(x)$, using a sequential procedure. We require that no matter which pairing is true, the probability of a correct selection must exceed a given constant $P$. We consider the degenerate case where the ratio $f(x)/g(x)$ can take only the values 0, 1, $\infty$. We examine both procedures which take only one observation at each sampling stage and those which take observations from all contending populations (non-contenders being eliminated from sampling) at each stage. For either type of sampling, we find an entire family of tests which achieve a minimum average sample size subject to the given constraints. These procedures use randomization to decide whether to stop or to continue sampling. Among these, there is one which minimizes the variance of the sample size. This is a pseudo-truncated, non-randomized (except at one stage) procedure.

**42. On a Multivariate Fisher's Z** (Preliminary Report). T. Cacoullos, University of Minnesota.

Let $R = (r_{ij})$ denote the sample correlation matrix obtained from $N$ observations on a normally distributed random vector $X = (X_1, \cdots, X_p)$ with population correlation matrix