DECISION PROCEDURES FOR FINITE DECISION PROBLEMS UNDER COMPLETE IGNORANCE[1]

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1. Introduction. This paper deals with a class of statistical decision problems considered as games against Nature. We are concerned with problems in which the number of states of Nature is finite, the set of decisions available to the statistician is finite, and the statistician is strictly without information as to which state of Nature will occur. Our purpose is to describe what we would consider to be a satisfactory method for selecting a decision in such problems. A list of properties, which we believe should characterize such a method is given. Modifying a method due to J. W. Milnor, we exhibit a procedure which has the required properties.

Related formulations of the problem have been treated by H. Chernoff [2] and J. W. Milnor [3].

The following is a more explicit characterization of the class of decision problems we will deal with.

We are given a probability space (S, P) where $S = \{s_1, s_2, \cdots, s_n\}$ is a set of states of Nature and P is a probability measure defined on all the subsets of S. A vector $\xi = (\xi_1, \xi_2, \cdots, \xi_n)$ is associated with S, where ξ is a point in Ξ, an $n - 1$ dimensional proability simplex, such that

$$P(\{s_j\}) = \xi_j \geqq 0 \qquad \text{for } 1 \leqq j \leqq n$$

$$P(\{s_i\} \cup \{s_j\}) = \xi_i + \xi_j \qquad \text{for } i \neq j$$

$$P(S) = \sum_{j=1}^{n} \xi_j = 1$$

We call ξ an *a priori* probability distribution on the elements of S. It is assumed that knowledge concerning ξ can be completely specified by a statement of the form $\xi \, \varepsilon \, \Xi_0$ where $\Xi_0 \subset \Xi$. If $\Xi_0 = \Xi$, we say we are in complete ignorance of ξ.

We are also given a set $D = \{d_1, d_2, \cdots, d_m\}$ of decisions exactly one of which must be selected prior to learning which state of Nature has occurred. We may regard the d_i as strategies in our game against Nature and will call the d_i pure strategies.

Also given is a real-valued function u defined on $D \times S$. $u(d_i, s_j)$ is a real number measuring the loss we incur when we choose d_i and state s_j occurs.

Then $P = (S, D, u)$ is a decision problem, which can be regarded as a finite game against Nature. The problem P can be identified with the loss matrix A,

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