ON A MEASURE OF TEST EFFICIENCY PROPOSED BY R. R. BAHADUR[1]

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1. Introduction. In [4] R. R. Bahadur has shown "that the study (as random variables) of the levels attained when two alternative tests of the same hypothesis are applied to given data affords a method of comparing the performances of the tests in large samples." This method of comparison produces as an end result a measure of asymptotic relative performance which in this paper is called the "Bahadur efficiency."

Both Bahadur [4] and this author [9] have pointed out that the Bahadur efficiency is in general only an approximate measure of asymptotic relative performance. In Section 2 we introduce the Bahadur efficiency by giving sufficient conditions for the measure to be exact. These conditions are generalizations of the conditions given by Bahadur ([4], p. 282).

In Section 3 we show that the conditions required to compute the Bahadur efficiency are much less restrictive than the discussion in [4] might indicate. A very general set of sufficient conditions is given, and the heuristic justification given in [4] by Bahadur for his method is sketched to show the modifications in argument required by this generalization.

In Section 4 a set of sufficient conditions easily applied in practice (but more restrictive than the conditions of Section 3) are given. These conditions are similar to those defining a "standard sequence" in [4], but are relaxed to allow general rates of convergence $L(n)$ rather than the more restrictive rate $L(n) = n$ assumed in [4]. An example of test comparison when $L(n) = (\log n)^{\frac{1}{2}}$ is discussed. Finally, we illustrate the "approximate" character of Bahadur efficiency by finding the Bahadur efficiency of two equivalent tests.

2. A special case of test comparison. For the duration of this paper it is understood that by the general problem of asymptotic test comparison we mean the following: We are given a set of probability measures $\{P_\theta\}$, $\theta \varepsilon \Omega$, defined over an arbitrary space S of points s. For $\Omega_0 \subset \Omega$, H is defined to be the hypothesis that $\theta \varepsilon \Omega_0$. To test H we have two sequences of test statistics $\{T_n^{(1)}\}$ and $\{T_n^{(2)}\}$, $n = 1, 2, \cdots$; we wish to compare the performances of the (asymptotically) optimal tests based on $\{T_n^{(i)}\}$, $i = 1, 2$, as $n \to \infty$ in the hope that some idea of the relative performance of these tests for any value of n may be gained.

Bearing this general context in mind, consider now a special case of our problem.

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