BAYESIAN ESTIMATION IN MULTIVARIATE ANALYSIS

By Seymour Geisser[1]

National Institute of Arthritis and Metabolic Diseases

1. Introduction. In this paper the Bayesian approach to multivariate analysis taken by Geisser and Cornfield [6] and Geisser [5] is extended and given a more comprehensive treatment. Most of the classical multivariate estimation problems are here considered from the Bayesian standpoint.

In Section 2, Bayesian estimation procedures are obtained for: a vector mean, linear combinations of the elements of a vector mean; simple and partial variances; simple, partial, and multiple correlation coefficients. The posterior distributions of the canonical correlations and of the principal components are also discussed. Section 3 is devoted essentially to linear combinations of independent vector means when a common covariance matrix is assumed and also when the covariance structure is different for each population. For the general multivariate linear hypothesis we demonstrate, in Section 4, that the joint Bayesian posterior region for the elements of the regression matrix is equivalent to the usual confidence region for these parameters. Further the joint predictive density of a set of future observations generated by the linear hypothesis is also obtained thus enabling one to specify the probability that a set of future observations will be contained in a particular region based only on previous data.

Essentially no new distributions are necessary for this Bayesian approach, since all the results are couched in terms of familiar densities. While some of the posterior regions are equivalent to well established confidence regions, others are not. They may differ either as to the degrees of freedom involved or because of the fact that certain "confidence distributions" are non-Bayesian inversions e.g. the correlation coefficient, Brillinger [2].

The prior densities or weight functions used here are basically those of [5] and [6] and purport to reflect to a large degree prior ignorance or relative diffuseness. These unnormed densities or weight functions presumably may be "justified" by various rules, e.g. invariance, conjugate families, stable estimation, etc., or heuristic arguments. Although their utilization here does not necessarily preclude other contenders which may also be conceived of as displaying a measure of ignorance, it is our view that no others at present seem to be either more appropriate or as convenient. The fact that their application yields in many instances the same regions as those of classical confidence theory is certainly no detriment to their use, but in fact provides a Bayesian interpretation for these well established procedures.

2. One Population. Let $\mathbf{x}_1, \cdots, \mathbf{x}_N$ be independent observations on a

Received 12 August 1964.

[1] Sponsored in part by the Mathematics Research Center, United States Army, Madison, Wisconsin, under Contract No.: DA-11-022-ORD-2059.