MAIN-EFFECT ANALYSIS OF THE GENERAL NON-ORTHOGONAL LAYOUT WITH ANY NUMBER OF FACTORS[1]

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1. Introduction. The analysis of variance of a non-orthogonal two-factor layout is described in detail by Scheffé [6] under the heading "Two-way layout with unequal cell-numbers;" a matrix derivation of the same results was given by Tocher [8]. A method of analysis for a non-orthogonal three-factor layout was given by Freeman and Jeffers [3]. Particular cases of four factor layouts, with some orthogonalities present, were treated e.g. by Pearce [4] and Clarke [1]. Using a different approach, several authors, like Corsten [2] and Stevens [7], have proposed iterative schemes for solving the normal equations.

No general practical method for analyzing a non-orthogonal p-factor layout seems to have been found so far. A survey of the present state of the problem is given by Pearce [5].

The main problem involved in the analysis is how to solve the normal equations i.e. how to project the vector of the observations on the linear hypothesis subspace. For the general p-factor layout, that subspace is the direct sum of a one-dimensional subspace, corresponding to the general mean, and of p subspaces corresponding to the p main-effects vectors. A direct solution of the normal equations requires the inversion of a matrix whose order is roughly equal to the total number of single-factor levels in the layout. Aside from the length of the calculations involved, this puts a definite limitation on the number of factors which can be handled, even by an electronic computer.

In this paper, it is shown (in Sections 4–7) that the normal equations can be solved by a stepwise transformation of the initial $p + 1$ subspaces into a set of $p + 1$ mutually orthogonal subspaces. The procedure, essentially analogous to the Gram-Schmidt method for orthogonalizing a set of vectors, consists of p steps. Each step requires the inversion of a matrix whose order is the number of levels of one of the factors. Therefore, provided the number of levels of each factor does not exceed the maximal order of the matrices which can be handled, there is practically no limitation on the number of factors.

In Section 9, the main-effect analysis based on the normal equations solution is described. In Section 10, it is shown that the analysis can be simplified in the presence of orthogonalities, and some special situations are treated, generalizing results already known for two, three and four factors.

It would, of course, be desirable to extend the method to the general non-orthogonal layout with, say, main-effects and two-factor interactions only.

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