ON THE ASYMPTOTIC THEORY OF FIXED-WIDTH SEQUENTIAL CONFIDENCE INTERVALS FOR THE MEAN

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1. Introduction. Let $x_1, x_2, \cdots$ be a sequence of independent observations from some population. We want to find a confidence interval of prescribed width $2d$ and prescribed coverage probability α for the unknown mean μ of the population. If the variance σ^2 of the population is known, and if d is small compared to σ^2, this can be done as follows. For any $n \geqq 1$ define

$$\bar{x}_n = n^{-1}\sum_1^n x_i, \qquad I_n = [\bar{x}_n - d, \bar{x}_n + d],$$

and choose a to satisfy

$$(2\pi)^{-\frac{1}{2}}\int_{-a}^{a} e^{-u^2/2}\, du = \alpha.$$

Then for a sample size n determined by

(1) $$n = \text{smallest integer} \geqq (a^2\sigma^2)/d^2,$$

the interval I_n has coverage probability

$$P(\mu \,\varepsilon\, I_n) = P(\sqrt{n}|\bar{x}_n - \mu|/\sigma \leqq d\sqrt{n}/\sigma).$$

Since (1) implies that $\lim_{d\to 0} (d^2n)/(a^2\sigma^2) = 1$, it follows from the central limit theorem that

$$\lim_{d\to 0} P(\mu \,\varepsilon\, I_n) = (2\pi)^{-\frac{1}{2}}\int_{-a}^{a} e^{-u^2/2}\, du = \alpha.$$

We shall be concerned with the case in which the nature of the population, and hence σ^2, is unknown, so that no fixed sample size method is available. Define

(2) $$v_n = n^{-1}\sum_1^n (x_i - \bar{x}_n)^2 + n^{-1} \qquad (n \geqq 1),$$

let $a_1, a_2, \cdots$ be any sequence of positive constants such that $\lim_{n\to\infty} a_n = a$, and define

(3) $$N = \text{smallest } k \geqq 1 \quad \text{such that} \quad v_k \leqq (d^2k)/a_k^2.$$

The object of the present note is to prove the following

Theorem. *Under the sole assumption that* $0 < \sigma^2 < \infty$,

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