

BOOK REVIEW

Correspondence concerning reviews should be addressed to the Book Review Editor, Professor William Kruskal, Department of Statistics, University of Chicago, Chicago 37, Illinois.

HARALD BERGSTRÖM, *Limit Theorems for Convolutions*. Almqvist and Wiksell, Stockholm, and John Wiley, New York, 1963. \$15.00, £5/4/4, 347 pp.

Review by D. A. DARLING

University of Michigan

The so-called "central limit problem" is quite old, and has been, by and large, the principal impetus in the development of probability theory over the past 200 years. This is the problem of finding necessary and sufficient conditions on a sequence of sums of independent and individually negligible random variables to converge in distribution, to find the class of possible limit distributions and conditions under which a given member obtains. Through the efforts of a great many men over the past 30 or 40 years the problem may be fairly said to be solved today. In 1949 Kolmogorov and Gnedenko published their profound and exhaustive treatise *Limit Distributions for Sums of Independent Random Variables*, which effectively and definitively closes the problem.

In an appendix to the English translation of this book, J. L. Doob states, "It is possible, although undesirable and unnatural, to write a book on limit distributions of sums of independent random variables with essentially the content of this book, but without the use of the name random variable anywhere in the text."

In the book under review Bergström does what Doob said would be possible. This achievement is all the more remarkable in that Bergström does not employ the powerful machinery of Fourier analysis either; thus this book is, if for no other reason, a work of great ingenuity and individuality, but also a marked regression from modern developments. In the preface the author says, "Since I present a new method I need not quote many earlier works on the subject," and he refers the reader to the book by Kolmogorov and Gnedenko for a "historical background"—indeed there is no bibliography and only five works are cited in the footnotes. There is another in one appendix in which "random variables" and "probabilities" are first mentioned; a second appendix, with another reference, consists of a few pages on the Lebesgue-Stieltjes integral.

The author considers convolutions of functions of bounded variation, normalized at points of discontinuity by the average of the right and left-hand limits. The first six chapters give definitions, a restricted development, with a few lapses, of Riemann-Stieltjes integration over the reals, a metrization of the space of functions considered by what the author calls the "Gaussian norm,"