STUDENT'S t IN A TWO-WAY CLASSIFICATION WITH UNEQUAL VARIANCES[1]

By Kay Knight Mazuy and W. S. Connor

Arthur D. Little and Texas A&M University

1. Introduction. Since "Student" discovered the random variable t and its distribution in 1908, the statistic t has been used to test for differences between the means of two populations. The use of this statistic for testing depends on assumptions of the normality of the two underlying distributions and on the equality of their variances. If either of these assumptions does not hold, the statistic is not valid. Scheffé [2] suggested a way whereby one can validly use the statistic t for two normal populations with unequal variances, and Cochran [1] discussed how to deal with heterogeneity of error variance associated with treatments in a randomized block experiment. The methods described by Scheffé and Cochran consist of forming independent estimates of the contrast to be tested and calculating t from them. The method is generalized in this paper.

The generalization was suggested by an experiment which was designed to compare the skill of three technicians in reading pulse characteristics using two testing devices. Inspection of the data suggested that the technicians differed appreciably in their abilities to repeat readings and that one of the devices gave more reproducible readings than the other. Accordingly, the present study was undertaken.

This study investigates the two-way classification with n observations per cell, where each cell is determined by one member from each class. It assumes that an observation is a realization of a random variable which is expressed as a sum of fixed effects, the overall mean and effects for the members which determine the cell, and a random error. With respect to the fixed effects, two models are considered: one without interaction and the other with interaction. The random errors are assumed to be statistically independent and normally distributed with means zero. A variance is associated with each member of each class, and the variance of the random errors in a cell is the sum of the variances associated with the members which determine the cell. Accordingly, the random variables for all cells have the same variance within a cell, but differ from cell to cell as determined by simple additive constraints. In order to use Student's t for testing contrasts among effects, it is necessary to find error contrasts which have the same variance as the contrast to be tested and are statistically independent of it and each other. The maximum number of such contrasts is determined and a method for finding them is presented.

Received 11 February 1963; revised January 1965.

[1] Much of this study was conducted under the sponsorship of the Army Research Office (Durham), Grant No. DA-ARO(D)-31-124-G266 while the authors were with the Research Triangle Institute.