DISTINGUISHING A SEQUENCE OF RANDOM VARIABLES FROM A TRANSLATE OF ITSELF

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1. Introduction. Suppose $X = \{X_1, X_2, \cdots\}$ is a sequence of independent and identically distributed random variables and $a = \{a_1, a_2, \cdots\}$ is a numerical sequence, a_n representing the error in centering X_n. When are the sample paths of X and $X + a$ distinguishable?

We can distinguish X and $X + a$ with probability one if a is so big that $\sum a_n^2 = \infty$. If $\sum a_n^2 < \infty$ and X has finite information (see Equation (1)) then we cannot distinguish. Conversely if we cannot distinguish for all a with $\sum a_n^2 < \infty$ then X has finite information. For X with finite information we can distinguish if and only if $\sum a_n^2 = \infty$. The latter statement becomes false for any wider class than the finite information class.

Here X is said to have finite information ($I < \infty$) if the common distribution F has a positive (a.e.) and (locally) absolutely continuous density φ and

$$\int_{-\infty}^{\infty} (\varphi')^2/\varphi < \infty. \tag{1}$$

Fisher [2] called the quantity in (1) the information, or intrinsic accuracy. It is denoted by $I = I(F)$.

We briefly mention an application to a quantization problem. Following J. Feldman [1] one can produce examples of quasi-invariant (qi) distributions in l_2: Construct a product measure λ on sequence space whose translates λ_a for $a \varepsilon l_2$ are all equivalent measures. Such a predistribution λ gives rise to a qi distribution on l_2 [1]. Theorem 1 gives, in particular, the exact class of such λ having identical one-dimensional marginals—namely those with finite information. Part of this result was obtained by Feldman who was concerned with more general situations. J. R. Klauder and J. McKenna have recently obtained very general classes of qi distributions in their work on continuous representations of l_2 [4].

Returning to the statistical setting, we say we can distinguish X and $X + a$ if there is a set E of sequences for which

$$P\{\Omega - E \mid X\} = P\{E \mid X + a\} = 0,$$

where Ω is the set of all sequences. This means that the measures μ and μ^a induced by X and $X + a$ respectively,

$$\mu(A) = P\{A \mid X\}, \qquad \mu^a(A) = P\{A \mid X + a\},$$

are singular ($\mu \perp \mu^a$).

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