CYCLIC DESIGNS[1]

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1. Introduction. Cyclic designs are incomplete block designs consisting in the simplest case of a set of blocks obtained by cyclic development of an initial block. More generally, a cyclic design consists of combinations of such sets and will be said to be of size (n, k, r), where n is the number of treatments, k the block size, and r the number of replications.

It is well known (e.g. Bose and Nair [1]) that cyclic development of a suitably chosen initial block is one of the methods of generating designs with a high degree of balance in the arrangement of the treatments such as balanced incomplete block (BIB) designs and partially balanced incomplete block designs with two associate classes (PBIB(2) designs). Again, the cyclic type is a rather junior partner among the five types into which Bose and Shimamoto [2] classify PBIB(2) designs. The emphasis in these and many related papers has been understandably on the number of associate classes, the cyclic aspect being incidental. In the present article we proceed in opposite fashion putting the cyclic property first. It will be shown how cyclic designs may be systematically generated and how the non-isomorphic designs of given size may be enumerated and constructed. All such designs are PBIB designs but may have up to $\frac{1}{2}n$ associate classes. For $n \leqq 15$ and $k = 3, 4, 5$, tables of the most efficient cyclic designs are presented and comparisons with BIB and PBIB(2) designs are made.

Points which make cyclic designs attractive are:

(i) *Flexibility.* A cyclic design of size (n, k, ik) exists for all positive integers n, k, i. If n and k have a common divisor d then a "fractional set" of size $(n, k, k/d)$ exists corresponding to each d. Fractional sets may be combined with designs of size (n, k, ik) to form fresh designs, or used by themselves especially if n is large. Thus there are cyclic designs for many sizes (n, k, r) for which no PBIB(2) design is available, but the reverse may also happen.

(ii) *Ease of representation.* No plan of the experimental layout is needed since the initial block or blocks suffice.

(iii) *Youden type.* In view of their method of generation cyclic sets with $r = k$, and hence combinations of such sets, provide *automatic* elimination of heterogeneity in two directions.

(iv) *Analysis.* For cyclic designs the coefficient matrix of the normal equations is a circulix. The inverse matrix may therefore be obtained explicitly (as another circulix), thus making possible a general method of analysis. Questions of analysis

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