ON THE GENERALIZED MELLIN TRANSFORM OF A COMPLEX RANDOM VARIABLE AND ITS APPLICATIONS

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1. Introduction. The Mellin transform

$$h(s) = E[X^s] \tag{1.1}$$

of a real positive random variable X is a useful tool to treat products

$$Y = A \cdot X_1 \cdots X_n \tag{1.2}$$

of independent positive random variables $X_1, X_2, \cdots, X_n$, A being a positive constant. It can also be used to treat products of powers

$$W = A \cdot X_1^{a_1} \cdot X_2^{a_1} \cdots X_n^{a_n} \tag{1.3}$$

where $a_1, a_2, \cdots, a_n$ are real (see [2], [3], [5], [6], [9]).

This Mellin transform is not as useful in cases for which X_k take both positive and negative values or complex values. W. M. Zolotariow [10] has given a tool to treat products of real (not necessary positive) random variables; this tool is not useful in cases when the factors are complex. P. Lévy has given a tool to treat products (1.2) of complex random variables (see [7]); this tool is not as useful for products (1.3) with a_k real.

In this paper a generalization of the Mellin transform (1.1) is given in such a way that it will be useful to treat products (1.3) where $X_1, X_2, \cdots, X_n$ are complex random variables for which $P\{X_k = 0\} = 0$, i.e. taking values in the set G^* of non-zero complex numbers, and a_k being real.

Under multiplication (1.2) the set G^* of non-zero complex numbers is an Abelian locally compact group isomorphic to the direct product $\mathcal{R} \times T$, where $\mathcal{R}$ is the multiplicative group of positive real numbers, which is isomorphic to the additive group of real numbers, and T denotes the additive group of real numbers modulo 2π. Given this structure of G^* the natural transform of a complex random variable $Z = R \cdot e^{i\Phi}$ on G^* would be

$$h(t, n) = E[R^{it}e^{in\Phi}], \quad -\infty < t < +\infty, n = \cdots, -1, 0, 1, \cdots. \tag{1.4}$$

(On this subject see [8], p. 141, [1], p. 73, [4], pp. 166–167).

The integral transform (1.4) does not suffice in cases where products (1.3) are treated with a_k being real but not necessary integer. In such a case it is more convenient to treat probability distributions on the set G being the Riemann surface of the function $w = \log z$. Under multiplication (1.3) the set G is isomorphic to the direct product $\mathcal{R} \times \mathcal{R}$, and that is why the natural transform of a probability distribution on G would be

$$h(t, v) = E[R^{it}e^{iv\Phi}], \qquad -\infty < t < +\infty, -\infty < v < +\infty. \tag{1.5}$$

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