SAMPLING ENTROPY FOR RANDOM HOMOGENEOUS SYSTEMS WITH COMPLETE CONNECTIONS

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In this note we derive the asymptotic behaviour of the sampling entropy for random homogeneous systems with complete connections with a finite set of states.

1. Let $X = (i)_{1 \leqq i \leqq r}$ be a finite set and W an arbitrary set. For every $i \varepsilon X$ let u_i be a mapping of W into itself and P_i a real-valued function defined on W such that $P_i \geqq 0$, $\sum_{i=1}^{r} P_i = 1$. We put $u_{i_1 \cdots i_n} = u_{i_n} \circ \cdots \circ u_{i_1}$ for $i_k \varepsilon X$, $1 \leqq k \leqq n$.

After [6] the mappings u_i and the functions P_i determine a random homogeneous system with complete connections; this concept contains as particular cases the simple or multiple chains with complete connections ([2], [10]), the chains of infinite order ([2], [3]), the stochastic models for learning ([1]) and the random automata ([9]). For every $c \varepsilon W$ there exist [6] a probability space $(\Omega, \mathcal{K}, \mathcal{P}_c)$ and a sequence of random variables $(\xi_n)_{n \varepsilon N^*}$, $N^* = \{1, 2, \cdots\}$, defined on Ω and with values in X, such that

$$\mathcal{P}_c(\xi_1(\omega) = i) = P_i(c)$$

$$\mathcal{P}_c(\xi_{n+1}(\omega) = i \mid \xi_n(\omega) = i_n, \cdots, \xi_1(\omega) = i_1) = P_i(u_{i_1 \ldots i_n}(c))$$

for any $n \varepsilon N^*$, $i \varepsilon X$, $(i_1 \cdots i_n) \varepsilon X^{(n)}$, where $X^{(n)}$ is the nth cartesian product of the set X.

For every $2 \leqq l \varepsilon N^*$, $(i_1 \cdots i_l) \varepsilon X^{(l)}$ let $P_{i_1 \ldots i_l}$ be the function defined on W by the relation

$$P_{i_1 \ldots i_l}(c) = P_{i_1}(c) P_{i_2}(u_{i_1}(c)) \cdots P_{i_l}(u_{i_1 \ldots i_{l-1}}(c)).$$

For every l and $n \varepsilon N^*$, $(i_1 \cdots i_l) \varepsilon X^{(l)}$ let $P_{i_1 \cdots i_l}^{(n)}$ be the function defined on W by the relations

$$P_{i_1 \cdots i_l}^{(n)} = P_{i_1 \cdots i_l}, \qquad \text{if } n = 1,$$

$$P_{i_1 \cdots i_l}^{(n)}(c) = \sum_{i=1}^{r} P_i(c) P_{i_1 \cdots i_l}^{(n-1)}(u_i(c)), \qquad \text{if } n > 1.$$

We have [6]

$$P_{i_1 \cdots i_l}^{(n)}(c) = \mathcal{P}_c(\xi_n(\omega) = i_1, \cdots, \xi_{n+l-1}(\omega) = i_l).$$

We set

$$a_n = \sup |P_i(u_{i_1 \cdots i_n}(c')) - P_i(u_{i_1 \cdots i_n}(c''))|$$

the upper bound being taken over all $c', c'' \varepsilon W$, $i \varepsilon X$, $(i_1 \cdots i_n) \varepsilon X^{(n)}$.

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