ON THE EXTRAPOLATION OF A SPECIAL CLASS OF STATIONARY TIME SERIES

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In this paper, we consider a problem previously solved by P. A. Kozuljaev [1]. Let $y(t)\{t = 0, \pm 1, \pm 2, \cdots\}$ be a discrete stationary (in the wide sense) time series, with zero mean, unit variance, and independent samples. Thus,

$$E\{y(t)\} = 0, \quad E\{[y(t)]^2\} = 1, \quad E\{y(t)y(t + \tau)\} = 0, \quad \tau \neq 0.$$

For an arbitrary but fixed $m > 1$, form the new random variable

$$x(t, m) = m^{-\frac{1}{2}} \textstyle\sum_{i=1}^{m} y(t + \mathrm{i}). \tag{1}$$

Then $E\{x(t, m)\} = 0$, and $E\{x(t, m)x(t + \tau, m)\} = R(\tau, m)$, where

$$\begin{aligned} R(\tau, m) &= 1 - |\tau|/m, \qquad && \tau = 0, \pm 1, \pm 2, \cdots, \pm(m - 1) \\ &= 0, && \tau \geqq m. \end{aligned} \tag{2}$$

The problem considered by Kozuljaev is that of extrapolating the stationary sequence (1). That is, for each fixed pair of positive integers p and n, he has solved the problem of determining coefficients $a_1, a_2, \cdots, a_n$ such that the variance

$$\mu(a_1, a_2, \cdots, a_n; m, p) = E\{(x(t + p, m) - \textstyle\sum_{i=1}^{n} a_i x(t + 1 - i, m))^2\} \tag{3}$$

is minimized. If $a_1, a_2, \cdots, a_n$ are chosen to minimize (3), then

$$\tilde{x}(t + p, m) = \textstyle\sum_{i=1}^{n} a_i x(t + 1 - i, m)$$

is the minimum variance (linear) estimate of $x(t + p, m)$, based on $x(t, m)$, $x(t - 1, m), \cdots, x(t - n + 1, m)$, and (3) is the variance of the estimate. For each m, n, and p Kozuljaev has determined the unique set of coefficients which minimizes (3), and has computed the minimum variance. He shows that the coefficients satisfy a certain linear algebraic system, given below, and solves this system by Cramer's rule. However, the evaluation of the determinants which arise in this method involves a large amount of labor, with the result that Kozuljaev's paper contains a series of some forty-four theorems, all of which are directed at solving this linear system, and calculating the resulting minimum variance.

In this paper, we give a different method of deriving Kozuljaev's results, based on considering the coefficients to be part of a sequence which is the solution of a certain difference equation. This viewpoint leads to a method of solution of the extrapolation problem, for this special case, which is considerably less involved than Kozuljaev's method.

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