A NOTE ON INCOMPLETE BLOCK DESIGNS WITH $b = v$[1]

By M. Bhaskar Rao

University of Bombay

Let N be the incidence matrix of a binary design with $b = v$, where b is the number of blocks and v is the number of treatments. Let each treatment be replicated r times and let the jth block size be k_j ($j = 1, 2, \cdots, v$). Damaraju Raghavarao stated a theorem in [2] that in a SUB (Symmetrical Unequal Block) arrangement with two unequal block sizes k_1 and k_2 and $b = v$, r lies in the open interval (k_1, k_2). In this note we will show that such arrangement does not exist.

Theorem . *If A is a non-singular matrix of order v such that* (i) $AE = cE$, (ii) $AA'E = dE$ *where c and d are scalars and E is the vector with v elements all unity, then* $A'E = (d/c)E$.

The proof is obvious and is omitted.

We know that in a SUB arrangement [2], with $b = v$,

$$NE = rE,$$

$$NN'E = [r + (v - 1)\lambda]E$$

where λ is the number of times any two treatments occur in the blocks. By the above theorem we get

$$N'E = [(r + (v - 1)\lambda)/r]E$$

which shows that all block sizes are equal, hence $k_j = r$ ($j = 1, 2, \cdots, v$). Therefore, N is a balanced incomplete block design. Thus we have, in a SUB arrangement with different block sizes, $b > v$.

Now we can define symmetrical balanced incomplete block design in the following way. It is a design in which v objects are arranged in v sets of k distinct objects such that between any two sets there are $\lambda(<k)$ objects in common.

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REFERENCES

[1] Chakrabarti, M. C. (1962). *Mathematics of Design and Analysis of Experiments*. Asia Publishing House, Bombay.

[2] Raghavarao, Damaraju (1962). Symmetric unequal block arrangements with two unequal block sizes. *Ann. Math. Statist.* **33** 620–33.

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