ADMISSIBILITY AND BAYES ESTIMATION IN SAMPLING FINITE POPULATIONS. III

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1. Introduction. In the Part I of this paper in Theorem 4.1 the Horvitz-Thomson estimate (H.T. estimate for short) of the population total was shown to be admissible in the class of all unbiased estimates. The restriction of unbiasedness was removed in Part II, but there the estimate shown to be admissible in the entire class, is different from the H.T. estimate. In Section 9 of Part I however the H.T. estimate was shown to be inadmissible in the entire class if the sampling design was not of fixed sample size, as defined there. Now in this part of the paper it is shown that for any sampling design of fixed sample size, the H.T. estimate is admissible in the class of all estimates satisfying a certain "regularity" (refer to Theorem 3.1) condition. This result, thus is a generalization of the Theorem 8.1 in Part I, where the H.T. estimate was proved to be admissible in the class of all linear estimates. As in Theorem 8.1 of Part I the present result is proved for a more general class of estimates of which the H.T. estimate is a particular case. Actually, for this general class but *excluding* the H.T. estimate, the admissibility is established following an argument due to the referee, among all 'measurable' estimates, thus relaxing the above referred to conditions of 'regularity.' In this connection we refer to Theorems 4.2 and 4.3.

One may note that the results, in this part of the paper are weaker than the result proved in the Part II, in the sense that they need the regularity or measurability conditions for their validation; and they are true for the fixed sample-size designs only, while the result of Part II is true regardless of any such restrictions.

ADDED AT PROOF STAGE: It is now clear to the author that due to a property of Laplace Transforms the results of this paper are valid *without* any reference to the *regularity* condition. However this and the measurability condition in this paper would be discussed in a subsequent publication.

2. Notation. The notation followed here is the same as formulated in Section 2 of Part I with the slight modification as adopted in Section 2 of Part II. The definitions and preliminaries in Section 2 of Part I are also applicable to the following discussion.

3. Admissibility of the estimate. As in Theorem 8.1 of Part I we shall prove the admissibility for a more general estimate of which the Horvitz-Thomson estimate is a particular case. We now denote the estimate by $\hat{e}(s, x)$. In Theorem 8.1 of Part I $\hat{e}(s, x)$ was defined by $\hat{e}(s, x) = \sum_{r \varepsilon s} b_r x_r$, where the coefficients b_r satisfy (i) $b_r \geqq 1$, $r = 1, 2, \cdots, N$, and (ii) $\sum_1^N 1/b_r = m$. Retaining condition

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