KENNETH S. MILLER, *Multidimensional Gaussian Distributions*. John Wiley and Sons, Inc., New York, 1964. $9.50. viii + 129 pp.

Review by HAROLD RUBEN

University of Minnesota

Two things should be said about this book. (1) The title is a misnomer. Of a total of 112 pages of text, 42 pages are devoted to a very special distribution associated with the multivariate Gaussian distribution, namely, the multivariate Rayleigh distribution. This is clearly the heart of the book. Only 36 pages, of which 17 are concerned nominally with Gaussian noise, actually deal with the subject named in the title. The remaining pages form an unclassifiable group concerned with various elementary topics which are not inherently connected with Gaussian distributions. A breakdown of this group results in the following list: diagonalization of a quadratic form, covariance matrices, inversion of a partitioned matrix, transformation of Cartesian to polar coordinates in n-space, the Fourier inversion formula and, finally, least squares estimation. (2) The discussion on the Gaussian distribution is far from connected, either spatially or logically. The same applies with yet greater force to the discussion on least squares estimation which does not always appear under that title and is in addition unduly repetitious, not to say downright muddled.

The p-variate Rayleigh distribution, as defined by the author, is the joint distribution of p correlated χ (not χ^2) variates, forming a Rayleigh random vector. More precisely, it is the distribution of the vector $(r_1, \cdots, r_p)$, where r_α^2 is the αth diagonal element of the matrix[1] $\mathbf{A} = \sum_1^n \mathbf{x}_\alpha \mathbf{x}_\alpha'$ and the $\mathbf{x}_\alpha$ are independent p-component normal vectors with arbitrary expectation vectors $\boldsymbol{\mu}_\alpha$ and a common positive definite covariance matrix $\mathbf{\Sigma}$. Some light is thrown on the distribution by noting that r_α^2 is a multiple of a (in general non-central) χ^2 variate, and further that in principle (but presumably only in principle) the joint distribution of the r_α^2 could be obtained by integrating out the "crossproduct" variates in the non-central Wishart distribution of $\mathbf{A}$, first studied apparently by T. W. Anderson [1], [2]. (A warning is in order here. The author's definition of the multivariate Rayleigh distribution (p. 27) is sloppy and strictly speaking vacuous. That 'definition' imposes normality on each of the p n-component vectors obtained by taking corresponding components of the $\mathbf{x}_\alpha$, and imposes only independence on the $\mathbf{x}_\alpha$ themselves.) The density functions of the distribution for $p = 1, 2, 3$ and for special values of the $\boldsymbol{\mu}_\alpha$ (mostly $\boldsymbol{\mu}_\alpha = \mathbf{0}$), as well as for general p with $\mathbf{\Sigma}$ a continuant and $\boldsymbol{\mu}_\alpha = \mathbf{0}$, are given in six theorems. A seventh theorem gives the density in symbolic form for arbitrary p, $\boldsymbol{\mu}_\alpha$ and $\mathbf{\Sigma}$. Three theorems deal respectively with the densities of the product of norms, the inner product and the angle between two correlated Gaussian vectors, and a further three theorems

[1] Here, as elsewhere in this review, I am adapting the author's notation to conform to common statistical usage.