ON THE PROBLEM OF TESTING LOCATION IN MULTIVARIATE POPULATIONS FOR RESTRICTED ALTERNATIVES[1]

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0. Introduction. In a comprehensive paper Bartholomew [2] treats the problem of testing equality of means in normal populations versus ordered and partially ordered alternatives. The application of the likelihood ratio (LR) principle to derive the test statistic leads to minimize a convex quadratic form subject to linear constraints. The author uses a theorem by van Eeden [7] to solve this problem but remarks that this could also be done by means of quadratic programming methods. This remark is the starting point to our investigation.

It is easy to see that the setup of Bartholomew is a special case of the following problem: Let $\mathbf{X}$ be a p-dimensional random vector, multivariate normally distributed with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. The null hypothesis H, that the center of the distribution lies at a point $\mathbf{0}$, is to be tested versus the alternative K that it lies in the union of some regions R_i, where each R_i is the intersection of p halfspaces through $\mathbf{0}$. (Without loss of generality we choose $\mathbf{0}$ as the origin of the coordinate system.) We may restrict ourselves to the alternative that $\boldsymbol{\mu}$ lies in the positive orthant $K_0 : \boldsymbol{\mu} \geqq \mathbf{0}$ with at least one strict inequality ($\boldsymbol{\mu} > \mathbf{0}$ denotes that all components of the vector are positive) since the methods developed extend immediately if the orthant is replaced by the union of arbitrarily many R_i. The extension has been carried out in more details in Nüesch [5].

Bartholomew's results follow from ours by specializing to particular covariance matrices. (e.g. the covariance matrix corresponding to completely ordered parameters is a so-called type 2 matrix with entries $\rho_{ij} = 0$ for $|i - j| > 2$.)

In Section 1 quadratic programming methods are used to minimize a general convex quadratic form $f(\mathbf{y})$ subject to the constraints $\mathbf{y} \geqq \mathbf{0}$. In Sections 2 and 3 the test statistics and their distributions are derived for the cases of known and unknown covariance matrix, respectively.[3]

1. Solutions of the reduced quadratic programming problem. Let $\mathbf{C}$ be a positive definite $p \times p$ matrix ($\mathbf{C} > \mathbf{0}$), $\mathbf{q}$ a $1 \times p$ matrix of constants. The function

$$f(\mathbf{y}) = 2\mathbf{q}\mathbf{y} + \mathbf{y}'\mathbf{C}\mathbf{y} \tag{1.1}$$

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[3] Section 2 overlaps with results by Kudô [4], which were published after this paper was submitted. Kudô uses an intuitive geometrical argument to obtain the maximum likelihood estimates. This argument however does not carry over to the case of unknown covariance matrix as does our approach.