SOME GENERALIZATIONS OF DISTINCT REPRESENTATIVES WITH APPLICATIONS TO STATISTICAL DESIGNS[1]

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1. Introduction. If $S_1, S_2, \cdots, S_n$ are n sub-sets of a given finite set S, then we say that $(a_1, a_2, \cdots, a_n)$ is a system of distinct representatives (SDR) for the sets $S_1, S_2, \cdots, S_n$ if a_i belongs to S_i and all a_i's are distinct. The necessary and sufficient condition in order that the sets $S_1, S_2, \cdots, S_n$ possess an SDR is that the union of any k of the sets contain at least k distinct elements ([6], [8]). The concept of distinct representatives has been generalized in various directions with a wide field of applications ([5], [8], [9]). In this paper some further generalizations are given with applications to design of experiments.

2. Generalization.

Definition 2.1. If $S_1, S_2, \cdots, S_n$ are the n sub-sets of a given finite set S, then $(O_1, O_2, \cdots, O_n)$ will be called a $(m_1, m_2, \cdots, m_n)$ SDR if

(i) $O_i \subseteq S_i$,

(ii) $n(O_i) = m_i$, and

(iii) $O_i \cap O_j = \emptyset, i \neq j, = 1, 2, \cdots, n,$

where $n(O_i)$ is the number of elements in the set O_i.

If $m_1 = m_2 = \cdots = m_n = m$, the sets will be said to possess an m-ple SDR.

We can prove the following theorem on similar lines as Theorem 2.1 of [8].

Theorem 2.1. *A necessary and sufficient condition in order that $S_1, S_2, \cdots, S_n$ may possess a $(m_1, m_2, \cdots, m_n)$ SDR is that*

$$n(S_{i_1} \cup S_{i_2} \cup S_{i_3} \cup \cdots \cup S_{i_k}) \geqq \textstyle\sum_{j=1}^{k} m_{i_j},$$

$$1 \leqq i_1 < i_2 < \cdots < i_k \leqq n; \quad 1 \leqq k \leqq n.$$

3. Applications.

Lemma 3.1. *Given positive integers v, b, r and k such that $bk = vr$ and $v > k$ then there exists an equi-replicate binary incomplete block design in v treatments each replicated r times in b blocks of constant block size k.*

Theorem 3.1. *In every binary equi-replicate design (with column as blocks) of constant block size k such that $bk = vr$ and $b = mv$, the treatments can be rearranged into blocks, so that every treatment occurs in a row m times.*

Proof. Form the sets $S_1, S_2, \cdots, S_v$ where S_i is the set of all block numbers containing the treatment i. Now,

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Received 23 November 1964; revised 12 July 1965.

[1] This work was financially supported by a Junior Research Fellowship of the Council of Scientific and Industrial Research (Delhi).

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