ADMISSIBILITY OF CONFIDENCE INTERVALS

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1. Introduction. Hodges and Lehmann (1951) have shown that for a sample of n independent observations from a univariate normal population the sample mean is an admissible estimator of the parent mean. More general results have recently been proved by Farrell (1964) regarding the admissibility of estimators of the location parameter in a class of continuous frequency functions. The analogous question regarding confidence intervals is considered here, and the admissibility of a class of confidence intervals is proved for the location parameter in a wide class of continuous frequency functions which includes the normal and some other commonly occurring ones. A practically important application of the result is that the usual symmetrical confidence intervals for the mean of a normal population based on the 't' statistic are seen to be admissible whether the population variance is known or not.

Again, the general result which is proved for confidence intervals whose length may be any random variable distributed independently of the location parameter under estimation, also includes as a particular case the admissibility of certain well known confidence intervals of constant length, obtained by minimizing the length for a given confidence level. For this particular case, however, the result can be established under less restrictive assumptions, either by a direct proof or as a deduction from Farrell's results (1964).

2. Notation. In the following, X denotes a real random variable with a df involving a parameter θ which assumes values in a set Ω of the real line; $x_1, \cdots, x_n$ independent observations of X; and $x = (x_1, x_2, \cdots, x_n)$ a point in the sample space $\mathfrak{X}$; on $\mathfrak{X}$ and Ω is defined the Lebesgue measure, all sets being Lebesgue measurable; $a(x)$, $b(x)$, with or without subscripts denote measurable functions defined on $\mathfrak{X}$, and $(a(x), b(x))$ denotes the set of confidence intervals $[a(x) \leqq \theta \leqq b(x)]$. We define admissibility of confidence intervals as below:

Definition 2.1. A set of confidence intervals $(a(x), b(x))$ is said to be admissible if and only if, there exists no other set of confidence intervals $(a_1(x), b_1(x))$ satisfying

(i) $b_1(x) - a_1(x) \leqq b(x) - a(x)$ for almost all $x \varepsilon \mathfrak{X}$, and

(ii) $P(a_1(x) \leqq \theta \leqq b_1(x) \mid \theta) \geqq P(a(x) \leqq \theta \leqq b(x) \mid \theta)$ for all $\theta \varepsilon \Omega$, the strict inequality in (ii) holding for at least one $\theta \varepsilon \Omega$. The definition of admissibility for confidence intervals was formulated by Godambe (1961) but in his formulation, the strict inequality in (ii) was required to hold on a non-null set of Ω. We have slightly modified his definition to make it agree with the conventional concept of

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