BOUNDED LENGTH CONFIDENCE INTERVALS FOR THE p-POINT OF A DISTRIBUTION FUNCTION, III[1]

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1. Introduction. Let $0 < p < 1$. A number $\gamma_{p,F}$ is a p-point of the distribution function F if $F(\gamma_{p,F}) \geqq p$ and $F(\gamma_{p,F}-) \leqq p$. Given $L > 0$ and $0 < \alpha < 1$, a L-α bounded length confidence interval procedure operates successfully for F means that when sampling stops an interval of length not exceeding L is given which covers $\gamma_{p,F}$ with probability at least $1 - \alpha$. It is the purpose of this paper to give the construction of two L-α bounded length confidence interval procedures which operate successfully for all $F \varepsilon \mathbf{F}$. $\mathbf{F}$ will be the set of all distribution functions F such that $\epsilon_F > 0$. ϵ_F is defined below in (1.10).

Throughout we let $\{X_n, n \geqq 1\}$ be a sequence of independently and identically distributed random variables such that F is the distribution function of X_1. As we allow different choices of F, we indicate the choice when computing expectations by use of "F" as a subscript.

The procedures constructed are measurable functions of the random variables $\{X_n, n \geqq 1\}$. Two sets of functions $\{u_{i,n}, n \geqq 1\}$ and $\{v_{i,n}, n \geqq 1\}$, $i = 1, 2$, are constructed. Properties of the procedures are as follows:

(1.1) If $i = 1, 2$ and if $n \geqq 1$ then $u_{i,n}$ and $v_{i,n}$ are real valued Borel measurable functions defined on Euclidean n-space, and $u_{i,n}(x_1, \cdots, x_n) \leqq v_{i,n}(x_1, \cdots, x_n)$ for all $(x_1, \cdots, x_n)$ in n-space.

(1.2) If $i = 1, 2$ and if $n \geqq 1$ let random variables be defined by $U_{i,n} = u_{i,n}(X_1, \cdots, X_n)$ and $V_{i,n} = v_{i,n}(X_1, \cdots, X_n)$. There exist real numbers α and β such that $0 < \alpha < 1$ and $0 < \beta < 1$ and such that if $F \varepsilon \mathbf{F}$ then

$$P_F(\text{all } n \geqq 1, U_{i,n} \leqq \gamma_{p,F}) \geqq 1 - \beta;$$

$$P_F(\text{all } n \geqq 1, V_{i,n} \geqq \gamma_{p,F}) \geqq 1 - \alpha.$$

(1.3) If F is a distribution function and $\gamma_{p,F}$ is the unique p-point of F then if $i = 1, 2$,

$$P_F(\lim_{n\to\infty} (V_{i,n} - U_{i,n}) = 0) = 1.$$

(1.4) If $f: (-\infty, \infty) \to (-\infty, \infty)$ is strictly increasing, then for $i = 1, 2$, $n \geqq 1$, and all $x_1, \cdots, x_n$,

$$u_{i,n}(f(x_1), \cdots, f(x_n)) = f(u_{i,n}(x_1, \cdots, x_n));$$

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