ON THE BLOCK STRUCTURE OF CERTAIN PARTIALLY BALANCED INCOMPLETE BLOCK DESIGNS

By S. M. Shah

Sardar Vallabhbhai Vidyapeeth

1. Introduction. In an earlier paper [3], the author gave the upper bounds for the number of disjoint blocks in (i) semi-regular GD designs, (ii) certain PBIB designs with two associate classes having a triangular association scheme, (iii) certain PBIB designs with two associate classes having an L_2 association scheme and (iv) certain PBIB designs with three associate classes having a rectangular association scheme. Later on, the author [4] gave bounds for the number of common treatments between two blocks of the above-mentioned designs. In this paper, we generalise the author's [3] results and give conditions under which no two blocks of the above-mentioned designs are (i) disjoint or (ii) the same set.

2. Semi-regular GD designs. An incomplete block design with v treatments each replicated r times in b blocks of size k is said to be group divisible (GD) [2], if the treatments $v = mn$ can be divided into m groups, each with n treatments, so that treatments belonging to the same group occur together in λ_1 blocks and treatments belonging to different groups occur together in λ_2 blocks ($\lambda_1 \neq \lambda_2$). The primary parameters of such a design are $v = mn$, b, r, k, $n_1 = (n - 1)$, $n_2 = n(m - 1)$, λ_1, λ_2. They obviously satisfy the relations $bk = vr$, $(n - 1)\lambda_1 + n(m - 1)\lambda_2 = r(k - 1)$, $r \geqq \lambda_1$, $r \geqq \lambda_2$. Semi-regular GD designs [1] are further characterised by $r - \lambda_1 > 0$ and $rk - v\lambda_2 = 0$.

From Theorem 2.1 of [3], we deduce Theorem 2.1.

Theorem 2.1. *If in a semi-regular* GD *design*, $b = v - m + r$ *and* $v = 2k$, *where* k *is an odd integer, then no two blocks of this design are disjoint.*

Theorem 2.2. *If in a given block of a semi-regular* GD *design with* $b > v - m + 1$ *has* d *blocks having a given number* l ($\leqq k$) *of treatments common with it, then*

$$d \leqq b - 1 - [k(r - 1) - l(b - 1)]^2/Q,$$

where $Q = P + l^2(b - 1) - 2lk(r - 1)$, *and* $P = k^2[(v - k)\cdot(b - r) - (v - rk)(v - m)]/v(v - m)$. *Further, if* $d = b - 1 - [k(r - 1) - l(b - 1)]^2/Q$, *then* $[P - lk(r - 1)]/[k(r - 1) - l(b - 1)]$ *is an integer and the given block has* $[P - lk(r - 1)]/[k(r - 1) - l(b - 1)]$ *treatments common with each of the remaining* $(b - d - 1)$ *blocks.*

Proof. We number the blocks $B_1, B_2, \cdots, B_b$. Let x_i denote the number of treatments common between B_1 and B_i, $i = 2, 3, \cdots, b$. Let $x_i = l$ for $i = 2, 3, \cdots, (d + 1)$.

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