COMPARISONS OF SOME TWO STAGE SAMPLING METHODS[1]

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1. Introduction. The use of multistage sampling procedures has been of great value in providing a solution to the problem of estimating a parameter with a prescribed precision. There are several two-stage methods available so that either (A) the estimate of a parameter has a specified variance, or (B) a $(1 - \alpha)$ confidence interval placed on a parameter has a specified width. Of the methods available that provide a solution to (A) or (B), the techniques of Birnbaum and Healy [2] (henceforth called BH), Stein [11], and Graybill [6] appear easiest to apply. The purpose of this paper is to present a general result that holds under certain conditions for obtaining the expected sample size in Graybill's method and to compare results where feasible with the techniques of Stein and BH. A review of Graybill's theorem is given. Brief explanations of the applications of the three methods are presented when estimating the mean or the variance from a normal population.

2. The expected sample size using Graybill's method. Suppose w is the width of a confidence interval on a parameter ξ with confidence coefficient $1 - \alpha$. Suppose further that it is desired that the probability that w be less than d lie between β^2 and $2\beta - \beta^2$. The problem is to determine k, the number of observations, on which to base w.

The Graybill [6] technique will be described for a two-stage procedure. The first stage yields a random variable z from which is determined a sample size k on which to base the confidence interval of random width w. Suppose that the distribution of w depends on k and an unknown parameter θ (θ may be the parameter ξ). Suppose also there exists a function g such that the distribution of $Y = g(w; \theta, k)$ depends only on k (and not on the unknown parameter) and g is monotonic increasing in w for every k and θ. Then a function $f(k)$ may be obtained so that $P[Y < f(k)] = \beta$; $0 < \beta < 1$. Let the solution for $g(w; \theta, k) = f(k)$ for w be $w = h(\theta, k)$ such that $h(\theta, k)$ is monotonic increasing for every k and monotonic decreasing in k for every θ.

Let n be defined as a random variable such that $h(t(z), n) = d$; consequently k is the smallest positive integer such that $k \geqq n$ and $h(t(z), k) \leqq d$. Then the following inequality is true:

$$\beta^2 \leqq P(w \leqq d) \leqq 2\beta - \beta^2.$$

At this point an expression for $E(k)$ shall be presented.

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