LIMIT THEOREMS FOR STOPPED RANDOM WALKS II[1]

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1. Introduction. Throughout we suppose $\{X_n, n \geqq 1\}$ to be a stationary metrically transitive sequence of k-dimensional random (column) vectors such that the components of X_1 are all *positive* with probability one and such that $E(X_1^T X_1)^{\frac{1}{2}} < \infty$. We will use the superscript "T" to indicate transpose. Let $h(\cdot)$ be a real valued homogeneous function of degree one defined and continuous throughout Euclidean k-space. We assume that on the open first quadrant $Q = \{x \mid \min_{1 \leqq i \leqq k} x_i > 0\}$ that $h(x) > 0$ and that on Q, h has continuous positive first partial derivatives. We will let α be the column vector of first partial derivatives of h evaluated at μ. The assumptions of this paragraph will be used throughout the remainder of the paper without further comment.

If $n \geqq 1$, define $S_n = X_1 + \cdots + X_n$, $S_0 = 0$. We use the notation $h(S_n) = H_n$, $n \geqq 0$. We define $N(t)$ to be the number of values H_n, $n \geqq 1$, which are less than $t \geqq 0$. Then if $t \geqq 0$, with probability one, $N(t) < \infty$ and $\lim_{t\to\infty} \cdot N(t)/t = 1/h(\mu)$. See Farrell [2]. Without risk of ambiguity we may define for $t \geqq 0$, $H_t = h(S_{N(t)})$. From our definitions it follows that with probability one, $H_t < t$ for all $t > 0$.

In this paper we are interested in studying the continuous parameter process $X(\cdot)$ defined by $X(t) = t - H_t$ if $t \geqq 0$. In the case that $\{X_n, n \geqq 1\}$ is a sequence of independently and identically distributed real valued random variables Doob [1] showed by use of Cesàro averages the construction of a stationary Markov measure (stationary under translations) for the continuous parameter process in which the joint distribution of the spacings between m successive jumps is the joint distribution of $X_1, \cdots, X_m$, $m \geqq 1$. The spacing from $t = 0$ to the first jump has a different distribution which is uniquely determined by the requirement that the resulting process be stationary. This is clear from the results of Doob, op. cit.

It is the primary purpose of this paper to generalize this result for point processes $\{H_n, n \geqq 1\}$ defined above and constructed from sums of random variables having a stationary distribution. In several dimensions, unless h is linear, there is no corresponding result except in the limit as $t \to \infty$. That is to say, in many ways as n becomes large the two sequences of random variables, $\{H_n, n \geqq 1\}$ and $\{\alpha^T X_n, n \geqq 1\}$ look approximately the same. It is this fact of linearization that underlies the theorem stated at the end of this section. We are concerned with the construction of a stationary measure on K_+, defined below, such that the joint distribution of m successive jumps is the joint distribution of $\alpha^T X_1, \cdots, \alpha^T X_m$, valid if $m \geqq 1$. This stationary measure is gen-

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