ON THE EXACT DISTRIBUTIONS OF THE LIKELIHOOD RATIO CRITERIA FOR TESTING LINEAR HYPOTHESES ABOUT REGRESSION COEFFICIENTS

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1. Introduction. Wilks (1932) defined a number of likelihood ratio criteria for testing the equality of means, equality of variances and equality of covariances from several populations. These criteria are being very widely used in applied work for tests of significance in multivariate analysis. Wilks (1934) and Bartlett (1934) extended their use for testing some linear hypotheses about regression coefficients.

If $x_1, x_2, x_3, \cdots, x_N$ are a set of vector observations, with associated fixed vectors $z_1, z_2, z_3, \cdots, z_n$, where x_α is an observation from $N(\beta_{z\alpha}, \Sigma)$, and if the matrix β is partitioned such that $\beta = (\beta_1\beta_2)$ where β_1 has q_1 columns and β_2 has q_2 columns, then the likelihood ratio criterion λ, for testing the hypothesis that the matrix β_1 is equal to some given matrix, is given by

$$\lambda = |\Sigma_1|^{N/2}/(|\Sigma_2|^{N/2})', \tag{1.1}$$

where Σ_1 and Σ_2 (sum of products matrices) are the maximum likelihood estimates of $p \times p$ matrix Σ over the full range and over the restricted range under the hypothesis.

Wilks (1932) has also obtained the hth moment of the criterion $U = \lambda^{2/N}$. When $N - q_1 - q_2 = n$ and $q_1 = m$, Anderson shows that the hth moment of $U_{p,m,n}$ can be put in the form

$$M_h(U_{p,m,n}) = \prod_{i=1}^{p} \Gamma[\tfrac{1}{2}(n + 1 - i) + h]\cdot\Gamma[\tfrac{1}{2}(n + m + 1 - i)]/ \Gamma[\tfrac{1}{2}(n + 1 - i)]\cdot\Gamma[\tfrac{1}{2}(n + m + 1 - i) + h]. \tag{1.2}$$

Wilks (1935) obtained the distribution of $U_{p,m,n}$ in the form of a $(p - 1)$ fold multiple integral, which he was able to evaluate for $p = 1, 2$; $p = 3$ with $m = 3, 4$ and for $p = 4$ with $m = 4$ only.

Bartlett (1938) suggested χ^2-significance points for $-m \log U_{p,m,n}$. Wald & Brookner (1941) obtained an asymptotic expansion for the distribution of $(-2 \log \lambda)$ and it was modified into a new form by Rao (1948). Box (1949) has given a general method of obtaining the asymptotic distributions of such criteria. Consul (1965) has given another similar general method. However, all these methods provide approximate values only.

Anderson (1958) has shown that the distribution of $U_{p,m,n}$ is that of a product of a number of independent beta variates and by integrating their joint densities he obtains explicit expressions for the distributions of $U_{p,m,n}$ for $p = 1$,

Received 7 July 1965; revised 23 May 1966.