SOME LIMIT THEOREMS FOR NON-HOMOGENEOUS MARKOV CHAINS

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1. Introduction. This paper deals with the problem of finding (necessary or sufficient) conditions for the relative stability and for the strong relative stability of sums of random variables (rv) which form a non-homogeneous Markov chain; we obtain also some results for the sums of arbitrarily dependent rv.

The results obtained in this paper are of classical form, i.e. they come very close to those obtained for mutually independent rv ([1], [3]–[7]); these classical results themselves remain true for a very large class of non-homogeneous Markov chains ($\alpha_i > \lambda > 0, i \varepsilon I = (1, 2, \cdots)$) and some of them for arbitrarily dependent rv. In the same way we obtain new results for homogeneous Markov chains ($\alpha_i = \lambda > 0, i \varepsilon I$). These results contain as particular cases the analogous results for mutually independent rv ($\alpha_i = 1, i \varepsilon I$). This paper contains also some new results for mutually independent rv.

For Markov chains, we express our results by means of the *ergodic coefficient* α of a stochastic transition function [2]; in [9] there can be found several of its definitions and properties that we shall use here; we shall also use the concept of *p-quantile*, the properties of which may be found in [13].

A part of these results were announced in preliminary papers ([11], [12]).

2. Notations. Let $(\mathfrak{A}_i, \Sigma_i)$ be a measurable space, x_i the elements of $\mathfrak{A}_i$, A_i measurable sets, elements of the σ-algebra Σ_i ($i \varepsilon I$). If the sequence of rv $\{\xi_i\}$ is a Markov chain, let us consider that it has the stochastic transition functions $P_i(x_i, A_{i+1})$. Let α_i denote the ergodic coefficient of P_i; that is

$$\alpha_i = 1 - \sup_{x,y\varepsilon\mathfrak{A}_i, A\varepsilon\Sigma_{i+1}} |P_i(x, A) - P_i(y, A)|.$$

Set $\beta_n = \min_{1 \leqq i < n} \alpha_i$. Assume $\alpha_i > 0$ for each $i \varepsilon I$, because in many important formulae ([8]–[10])β_n appears in the denominator.

3. Definitions. $\{\xi_n\}$ is (a) *stable* (S); (b) *strongly stable* (SS); (c) *relatively stable* (RS); (d) *strongly relatively stable* (SRS) if there is some sequence of constants $\{d_n\}$ so that respectively (a) $\{\xi_n - d_n\}$ converges in probability to zero, (b) $\{\xi_n - d_n\}$ converges almost everywhere to zero, (c) $\{\xi_n/d_n\}$ converges in probability to one, (d) $\{\xi_n/d_n\}$ converges almost everywhere to one.

$\{\xi_n\}$ is (a) *normally stable* (NS), (b) *normally strongly stable* (NSS), (c) *normally relatively stable* (NRS), (d) *normally strongly relatively stable* (NSRS) if in the given definitions we may take $d_n = M\xi_n$, the expectation of ξ_n.

We set $\zeta_n = \sum_{i=1}^n \xi_i$.

Let us suppose that $\{\zeta_n\}$ is RS with constants $\{d_n\}$; the ξ_i/d_n ($1 \leqq i \leqq n \varepsilon I$)

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