STOPPING TIME OF A RANK-ORDER SEQUENTIAL PROBABILITY RATIO TEST BASED ON LEHMANN ALTERNATIVES*,[1]

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1. The problem. We consider sequential probability ratio tests based on ranks for the two sample problem. The hypotheses used in computing the probability ratio are that the two sampled populations are identical and that one of the sampled populations has a distribution function which is a specified power of the other sampled distribution function. It is shown that this procedure terminates with probability 1 and the moments of the stopping time are finite. These results apply to whatever populations are actually sampled.

The problem to be discussed was presented by Robert Berk in a letter of July, 1964 to I. Richard Savage. It has also appeared in the work of E. A. Parent (1965) with which J. Sethuraman was familiar. Again, it appears in an appropriate theoretical context in Hall, Wijsman and Ghosh (1965, p. 594). Miss Sarla D. Merchant considered the problem in her unpublished Florida State University Masters' thesis of 1962.

To be specific we are concerned with the following situation: (X_1, Y_1), (X_2, Y_2), $\cdots$ are independently and identically distributed bivariate random variables with a joint distribution $H(\cdot, \cdot)$ which has continuous marginal distributions $F(\cdot)$ and $G(\cdot)$. We wish to test the null hypothesis $H_0 : X, Y$ *are independent, and* $G = F$ against the alternative hypothesis $H_1 : X, Y$ *are independent, and* $G = F^A$ where $A > 0$, $A \neq 1$ is a known constant. At the nth state of experimentation the available information is the ranks of $(Y_1, \cdots, Y_n)$ among $(X_1, \cdots, X_n, Y_1, \cdots, Y_n)$. We shall use a sequential probability-ratio test based on ranks (see Savage and Savage (1965)). If the distribution of (X, Y) is $H(\cdot, \cdot)$ with marginals $F(\cdot)$ and $G(\cdot)$ and $S(A, H) = S(A, F, G) \neq 0$ (for definition, see (12)) it is shown in Section 3 (Theorem 3) that this sequential test terminates with probability 1 and that the moment generating function of the required sample size is finite.

2. Notations, test procedure, preliminaries and lemmas. When the experiment has proceeded up to the nth stage we have observed (X_1, Y_1), (X_2, Y_2), $\cdots$, (X_n, Y_n). Let the combined sample be denoted by $Z_1, Z_2, \cdots, Z_{2n}$ and the ordered combined sample by $Z_{n1}, Z_{n2}, \cdots, Z_{n2n}$. Let $F_n(\cdot)$ and $G_n(\cdot)$ be the

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