

ON THE ADMISSIBILITY OF INVARIANT ESTIMATORS OF ONE OR MORE LOCATION PARAMETERS¹

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0. Introduction. Many statistical estimation problems possess certain natural symmetries. The location parameter estimation problem is an important example. It is symmetric, or, to use the usual terminology, invariant with respect to translations of the sample space. This strongly suggests that the statistician should use an estimation procedure which also has the property of being invariant.

Until recently it seemed reasonable to expect that the best invariant estimator is a "good" estimator. In particular, it seemed reasonable to expect that it is admissible—that is, that no other estimator has a risk which is never larger and is sometimes smaller than the risk of the best invariant estimator. However, Stein (1956) gave an example of a very simple problem in which the best invariant estimator is inadmissible. Previously several authors had proven admissibility in different problems, and there has been much research in the area since then. Several references are contained in the bibliography of this paper.

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