A NOTE ON CONSTRUCTION OF PARTIALLY BALANCED INCOMPLETE BLOCK DESIGNS WITH PARAMETERS $v = 28, n_1 = 12, n_2 = 15$ AND $p_{11}^2 = 4$[1]

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It is well known [2] that there are exactly four different association schemes of PBIBD with the parameters $v = 28, n_1 = 12, n_2 = 15, p_{11}^2 = 4$. The problem then arises what possibilities does this offer for construction of designs. For $v = b$ one can clearly construct two sets of four distinct designs by forming blocks consisting of all first associates or all second associates of each treatment according to each association scheme. For $v < b$ the problem of construction of designs was not yet investigated. For $v > b$ it is shown here that no other association scheme but the triangular can be used to construct designs.

Let N be the incidence matrix of the design. Connor and Clatworthy [5] showed that:

$$|NN'| = rk\rho_1^{\alpha_1}\rho_2^{\alpha_2} \quad \text{with} \quad \alpha_1 + \alpha_2 = v - 1.$$

If $v \geqq b$ then $\rho_i = 0$ for $i = 1$ or $i = 2$ and $b \geqq v - \alpha_i$ where α_i is the multiplicity of the zero root.

For $n = 8$ let

$$\rho_1 = r + 4\lambda_1 - 5\lambda_2, \qquad \alpha_1 = 7;$$

$$\rho_2 = r - 2\lambda_1 + \lambda_2, \qquad \alpha_2 = 20.$$

Case 1. $\rho_1 = 0$. The first kind of the parameters of the designs have to satisfy the following equations:

$$r = -4\lambda_1 + 5\lambda_2; \tag{1}$$

$$r(k-1) = 12\lambda_1 + 15\lambda_2; \tag{2}$$

$$28r = bk. \tag{3}$$

Equations 1 and 2 give: $rk = 8\lambda_1 + 20\lambda_2$. Thus rk must be divisible by 4. Eliminating λ_1 from equations (1) and (2) we get $r(k+2) = 30\lambda_2$. In addition to this $b \geqq 21$ since the multiplicity of ρ_1, $\alpha_1 = 7$. Using this information it is easy to establish that there are just two sets of solutions satisfying the above conditions. They are:

(a) $v = 28; b = 21, r = 15, k = 20, \lambda_1 = 10, \lambda_2 = 11;$

(b) $v = 28, b = 21, r = 6, k = 8, \lambda_1 = 1, \lambda_2 = 2.$

Neither of these sets represents a design. First notice that it suffices to show

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