ADMISSIBILITY AND BAYES ESTIMATION IN SAMPLING FINITE POPULATIONS—IV

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1. Introduction. This paper, which we call Part IV, is a continuation of the previous papers on the same subject, Part I by Joshi and Godambe (1965) and Parts II and III by Joshi (1965). In the introductory section of Part III a note was added at the proof stage, that it was realized that the result proved in that part relating to the admissibility of the Horvitz-Thomson estimate was valid for the class of all measurable estimates, without any 'regularity' restrictions on the class. We give a clarification of this point here and also add a supplementary, though minor result, subsequently obtained, that even the measurability restriction is removed for the special cases of sample size $m = 1$ and 2.

Next we give new results obtained mainly by applying the method developed in Part III, regarding the admissibility of the well-known ratio estimates, and the regression estimate for finite populations. While one ratio estimate is found to be always admissible whatever the sampling design, in the class of all measurable estimates, the other ratio estimate is shown to be necessarily admissible only when the sampling design is of fixed size, and that too subject to a certain condition. The regression estimate is also shown to be not always admissible.

2. Notation. The same notation is followed as in the previous parts, as specified in Section 2, Part I and Section 2, Part II. The definitions and preliminaries in Section 2 of Part I also all apply.

3. Superfluity of regularity conditions. The question relates to (15) in Part III where the Cramér-Rao lower bound for the variance of an estimate is assumed to apply. By (14) in Part III, the probability density is that of m independently distributed normal variates. The Wolfowitz conditions (1947) which are sufficient for the Cramér-Rao inequality are considered and shown to be all satisfied for a normal density function in Problem 1 of Hodges and Lehmann (1951). Our case is slightly different in that the variances of the variates may be unequal. But it is easily seen that this makes no difference in regard to conditions (i) to (iv) stated in Problem 1 of Hodges and Lehmann (1951). It will therefore suffice to verify that the remaining condition (v) is also satisfied. For the density function $L(y/\theta)$ in (12) of Part III, this condition becomes:

(v) The expression $\int g(s, y)L(y/\theta)\, dy$ may be differentiated under the integral sign.

But this follows readily from the fact that by definition of $g(s, y)$ in (5) and (10) Part III, $[g(s, y)]^{\frac{1}{2}}$ is bounded for all y, by a quadratic expression in y, say $u(y)$, i.e.

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