FIXED SIZE CONFIDENCE ELLIPSOIDS FOR LINEAR REGRESSION PARAMETERS[1]

By Arthur Albert

ARCON Corporation

TABLE OF CONTENTS

1. Introduction. In [2], Chow and Robbins developed an asymptotic theory of fixed width sequential confidence intervals for the mean of a univariate population. In [3], Gleser applied their stopping rule to the problem of constructing fixed size confidence sets for linear regression parameters. Gleser's confidence regions are spheres, centered at the least squares estimate of the regression parameter. In order that his method be valid, a strong assumption must be made concerning the large sample behavior of the least squares estimator's covariance matrix. Specifically, it is assumed in [3], that

$$\lim_{n\to\infty} n\Sigma_n = \Sigma \tag{1.1}$$

exists and is non-singular where Σ_n is the covariance matrix of the l.s.e. The stopping rule depends, in fact, upon the eigenvalues of Σ. The confidence regions proposed here are the more conventional ellipsoidal ones (whose kernels are proportional to Σ_n^{-1}), and while we are at it, we develop the theory to include estimable functions of the regression vector. Having done so, it is a simple task to adapt our results to the task of constructing sequential tests of the general linear hypothesis.

Our methods require that the least squares estimator for the regression parameter and its associated residual error be updated constantly as each datum is taken. To facilitate the attendant computations, we supply algorithms which allow these quantities to be computed iteratively "in real time" (as the data are collected).

In the last section, we exhibit formulae for the exact coverage probabilities in the case of normally distributed residuals.

Received 23 August 1965; revised 11 May 1966.

[1] This research was sponsored in part by the Office of Naval Research Contract No. Nonr 4401(00). Reproduction in whole or in part is permitted for any purpose of the United States Government.