TOLERANCE AND CONFIDENCE LIMITS FOR CLASSES OF DISTRIBUTIONS BASED ON FAILURE RATE

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1. Introduction. A fundamental problem in statistical reliability theory and life testing is to obtain lower tolerance limits as a function of sample data, say $\underline{X} = (X_1, X_2, \cdots, X_n)$. That is, if X denotes the time to failure of an item with distribution F, then we seek a function $L(\underline{X})$ such that

$$P_F\{1 - F[L(\underline{X})] \geqq 1 - q\} \geqq 1 - \alpha.$$

We call $1 - q$ the population coverage for the interval $[L(\underline{X}), \infty]$ and $1 - \alpha$ the confidence coefficient. Also, we want $U(\underline{X})$ such that $P_F\{F[U(\underline{X})] \geqq q\} \geqq 1 - \alpha$. Related problems are those of obtaining confidence limits on moments and percentiles.

Parametric tolerance limits based on the normal and exponential distributions are well known [8], [9], [14]. Goodman and Madansky [10] examine various criteria for goodness of tolerance intervals and certain optimum properties of the usual exponential tolerance limits are demonstrated. Recently, a great deal of effort has been devoted to obtaining various confidence limits for the Weibull distribution. Dubey [6] obtains asymptotic confidence limits on $1 - F(T)$ and the failure rate for the class of Weibull distributions with nondecreasing failure rate. He also studies the properties of various estimators for Weibull parameters, [7]. Johns and Liberman [12] present a method for obtaining exact lower confidence limits for $1 - F(T)$ when F is the Weibull distribution with both scale and shape parameters unknown. Unlike Dubey, they do not require that the Weibull distribution in question have a nondecreasing failure rate. These confidence limits are obtained both for the censored and noncensored cases and are asymptotically efficient.

There exist distribution-free tolerance limits, [13], based on say, the kth order statistic X_k for certain values of q, α, k and sample size N. However, they have one unfortunate disadvantage. For given α, q, k there is a minimum sample size $N(\alpha, q, k)$ such that

$$P_F\{1 - F(X_k) \geqq 1 - q\} \geqq 1 - \alpha$$

is true only if $N \geqq N(\alpha, q, k)$. Hanson and Koopmans [11] obtain upper tolerance limits for the class of distributions with increasing hazard rate and lower

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