LIMIT THEOREMS FOR STOPPED RANDOM WALKS III[1]

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1. Introduction. This paper explores asymptotic properties of certain first passage problems in several dimensions. Throughout we consider a function h which is a homogeneous function of degree one in k variables defined throughout Euclidean k-space E_k. Under consideration are random walks constructed from a sequence of random k-dimensional column vectors $\{X_n, n \geqq 1\}$. Set $S_n = X_1 + \cdots + X_n$, $n \geqq 1$ and $S_0 = 0$. If $n \geqq 1$ let $H_n = h(S_n)$. Given $t > 0$ define a stopping variable $M(t)$ to be the least integer n such that $H_n \geqq t$, with $M(t) = \infty$ if for all $n \geqq 1$, $H_n < t$. Study of the asymptotic behavior of $M(t)$ as $t \to \infty$ in the multidimensional case was started in Farrell [8]. Related results have appeared in Bickel and Yahav [2]. A slightly different random variable, $M'(t)$, is used in Section 3. See (3.6).

As indicated above, although the random walk is k-dimensional the quantities of interest here are definable in terms of the one-dimensional point process $\{H_n, n \geqq 1\}$. It is the main purpose of this paper to show that results like an analogue of Blackwell's theorem in renewal theory still hold here. See [2], [3] and [6].

Some elementary results, given in Section 2, can be obtained whenever the random variable sequence $\{X_n, n \geqq 1\}$ obeys the strong law of large numbers (the limit need not be constant and this is noted in the statements of lemmas and theorems,) $\lim_{n\to\infty} S_n/n = \mu$. Usually in renewal theory one assumes $\mu = EX_1$. Unless explicitly stated in hypotheses this is not assumed here. To distinguish cases we write at the start of each theorem in parentheses the appropriate hypotheses.

In the cases where we assume $\{X_n, n \geqq 1\}$ are independently and identically distributed we will always assume the existence of finite first moments for the component random variables of X_1 and we will write $\mu = EX_1$. We will suppose throughout that the components of μ are positive (even in the case μ is allowed to be a random variable.) Problems for which $\mu \neq 0$ and this is not so can, by a rotation of coordinate axes, be brought to this form. We will call $\{x \mid \min_{1 \leqq i \leqq k} x_i > 0\} = Q$ the open first quadrant and will call the closure of Q the first quadrant (of E_k). We will always suppose that h is continuous as a function on Q but for special reasons discussed later we will suppose in Section 3 that if $x \not\in Q$ then $h(x) = 0$. This restriction is special to Section 3. Unless otherwise stated we will suppose that h is positive everywhere on Q. Some of our results use the assumption that h has continuous first partial derivatives throughout Q. We

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