# A FORMULA FOR THE PROBABILITY OF OBTAINING A TREE FROM A GRAPH CONSTRUCTED RANDOMLY EXCEPT FOR AN "EXOGAMOUS BIAS"[1]

By Hwa Sung Na and Anatol Rapoport

*University of Michigan*

**1. Introduction.** A general problem in the probabilistic theory of linear graphs can be stated as follows:

Given a randomly constructed linear graph $G(n, N)$ with $n$ nodes and $N$ links and a property of linear graphs $A$, what is the probability that $G(n, N)$ will have the property $A$ as a function of $n$ and $N$?

The phrase "randomly constructed" needs to be more precisely specified, for example, by describing the process of construction. One such process consists of selecting from the $\binom{n}{2}$ pairs of nodes a random sample of $N \leqq \binom{n}{2}$ pairs to be connected by links. Accordingly, the probability of having property $A$ will then be defined as the ratio of the number of distinct labelled graphs with $n$ nodes and $N$ links which have this property, to the total number of such graphs, namely $C(n, N) = \binom{\binom{n}{2}}{N}$.

In particular, if $A$ is the property of being a connected graph, it was shown by Erdös and Rényi (1960) that if

$$N = (\tfrac{1}{2})n \log_e n + an + o(n), \tag{1}$$

then, as $n$ and $N$ approach infinity, the probability that the randomly constructed graph is connected approaches

$$P(A) = \exp\{-e^{-2a}\}. \tag{2}$$

In other words, given $N$ and $n$, both sufficiently large and connected by equation (1), the probability that $G(n, N)$ is connected is approximately

$$\exp\{-ne^{-2N/n}\}. \tag{3}$$

Many situations can be represented as linear graphs, for example, acquaintance nets in which the nodes are people and a link represents the relation of being acquainted; word association nets, where the nodes are words and a link represents the property of being associated in some sense (syntactic, semantic, etc.). One can imagine such graphs being generated by a stochastic process of some sort. However, it is clearly improbable that in this process links are formed entirely at random. Biases can certainly be expected to influence the probabilities of con-

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