A METHOD FOR THE CONSTRUCTION OF SECOND ORDER ROTATABLE DESIGNS IN k DIMENSIONS[1]

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1. Introduction. Box and Hunter (1957) gave conditions under which designs for the exploration of response surfaces would be rotatable. Since the appearance of their paper, many methods have been developed for constructing rotatable designs. In particular, Draper (1960) presented the following method of constructing a second order rotatable design in k dimensions from a second order rotatable design in $(k - 1)$ dimensions.

If the N' points

$$(1) \qquad (x_{1u}, x_{2u}, \cdots, x_{(k-1)u}), \qquad u = 1, 2, \cdots, N',$$

form a second order rotatable arrangement in $(k - 1)$ dimensions, that is, the points of (1) are such that

$$(2) \qquad \textstyle\sum_{u=1}^{N} x_{iu}^2 = A \neq N', \qquad \sum_{u=1}^{N} x_{iu}^4 = 3\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = 3C,$$

where $(i \neq j)$ $i, j = 1, 2, \cdots, (k - 1)$ and all other sums of powers and products up to and including order four are zero, then the point sets

$$(3) \qquad \begin{array}{ll} (x_{1u}, x_{2u}, \cdots, x_{(k-1)u}, \pm b), & u = 1, 2, \cdots, N', \\ (\ 0,\ 0,\ \cdots,\ 0,\ \pm p), & \\ (\ 0,\ 0,\ \cdots,\ 0,\ \pm q), & \end{array}$$

where

$$b^2 = C/A,$$
$$p^2, q^2 = \{(A^2 - N'C) \pm [2C(3A^2 - N'C) - (A^2 - N'C)^2]^{\frac{1}{2}}\}/2A,$$

form a second order rotatable arrangement in k dimensions. The number of points in the derived design will be $N = 2N' + 4 + n_0$, where n_0 denotes the number of center points needed to make the arrangement into a design. Since p^2 and q^2 must be real and non-negative, it is required that

$$(4) \qquad 1 \leqq \phi \leqq 2, \qquad \text{where} \qquad \phi = (A^2 - N'C)^2/C(3A^2 - N'C).$$

Therefore, it is possible to form second order rotatable designs in k dimensions

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